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Grade 11th

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BACCALAUREATE SCHOOLS

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Function Definition and Notation

Mathematics and its applications abound with examples of formulas by which quantitative variables are related to each other. The language and notation of functions is ideal for that purpose. A function is actually a simple concept; if it were not, history would have replaced it with a simpler one by now. Here is the definition.

DEFINITION Function, Domain, and Range

A **function** from a set D to a set R is a rule that assigns to each element in D a unique element in R. The set D of all input values is the **domain** of the function, and the set R of all output values is the **range** of the function.

There are many ways to look at functions. One of the most intuitively helpful is the "machine" concept (Figure 1.9), in which values of the domain (x) are fed into the machine (the function f) to produce range values (y). To indicate that y comes from the function acting on x, we use Euler's elegant **function notation** $y = f(x)$ (which we read as "**y equals f of x**" or "**the value of f at x**"). Here x is the **independent variable** and y is the **dependent variable**.

A function can also be viewed as a **mapping** of the elements of the domain onto the elements of the range. Figure 1.10a shows a function that maps elements from the domain X onto elements of the range Y. Figure 1.10b shows another such mapping, but *this one is not a function*, since the rule does not assign the element x_1 to a *unique* element of Y.

EXAMPLE 1 Defining a Function

Does the formula $y = x^2$ define y as a function of x?

SOLUTION Yes, y is a function of x. In fact, we can write the formula in function notation: $f(x) = x^2$. When a number x is substituted into the function, the square of x will be the output, and there is no ambiguity about what the square of x is.

Now try Exercise 3.

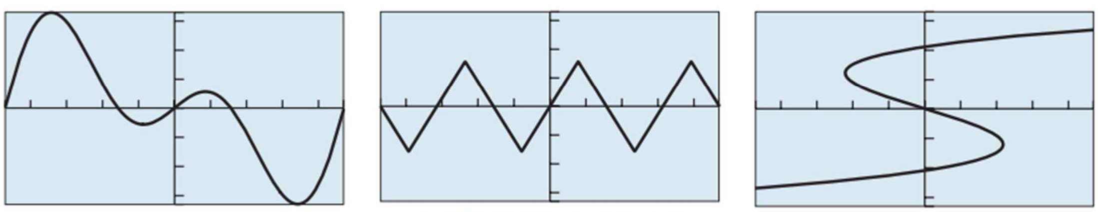
Another useful way to look at functions is graphically. The **graph of the function $y = f(x)$** is the set of all points $(x, f(x))$, x in the domain of f. We match domain values along the x-axis with their range values along the y-axis to get the ordered pairs that yield the graph of $y = f(x)$.

EXAMPLE 2 Seeing a Function Graphically

Of the three graphs shown in Figure 1.11, which is *not* the graph of y as a function of x? How can you tell?

SOLUTION The graph in (c) is not the graph of y as a function of x. There are three points on the graph with x-coordinate 0, so the graph does not assign a *unique* value to 0. (Indeed, we can see that there are plenty of numbers between -2 and 2 to which the graph assigns multiple values.) The other two graphs do not have a comparable problem because no vertical line intersects either of the other graphs in more than one point. Graphs that pass this *vertical line test* are the graphs of functions.

Now try Exercise 5.



$[-4.7, 4.7]$ by $[-3.3, 3.3]$
(a)

$[-4.7, 4.7]$ by $[-3.3, 3.3]$
(b)

$[-4.7, 4.7]$ by $[-3.3, 3.3]$
(c)

Figure 1.11 One of these is not the graph of y as a function of x. (Example 2)

Unless we are dealing with a model (like volume) that necessitates a restricted domain, we will assume that the domain of a function defined by an algebraic expression is the same as the domain of the algebraic expression, the **implied domain**. For models, we will use a domain that fits the situation, the **relevant domain**.

EXAMPLE 3 Finding the Domain of a Function

Find the domain of each of these functions:

(a) $f(x) = \sqrt{x + 3}$

(b) $g(x) = \dfrac{\sqrt{x}}{x - 5}$

(c) $A(s) = (\sqrt{3}/4)s^2$, where $A(s)$ is the area of an equilateral triangle with sides of length s.

SOLUTION

Solve Algebraically

(a) The expression under a radical may not be negative. We set $x + 3 \geq 0$ and solve to find $x \geq -3$. The domain of f is the interval $[-3, \infty)$.

(b) The expression under a radical may not be negative; therefore $x \geq 0$. Also, the denominator of a fraction may not be zero; therefore, $x \neq 5$. The domain of g is the interval $[0, \infty)$ with the number 5 removed, which we can write as the *union* of two intervals: $[0, 5) \cup (5, \infty)$.

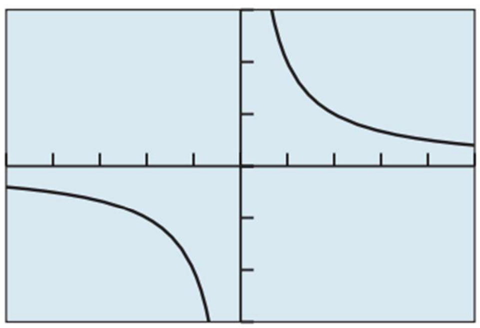
(c) The algebraic expression has domain $(-\infty, \infty)$, but the behavior being modeled

EXAMPLE 4 Finding the Range of a Function

Find the range of the function $f(x) = \dfrac{2}{x}$.

SOLUTION

Solve Graphically The graph of $y = \dfrac{2}{x}$ is shown in Figure 1.14.



$[-5, 5]$ by $[-3, 3]$

Figure 1.14 The graph of $y = 2/x$. Is $y = 0$ in the range?

It appears that $x = 0$ is not in the domain (as expected, because a denominator cannot be zero). It also appears that the range consists of all real numbers except 0.

Confirm Algebraically We confirm that 0 is not in the range by trying to solve $2/x = 0$:

$$\frac{2}{x} = 0$$

Continuity

One of the most important properties of the majority of functions that model real-world behavior is that they are *continuous*. Graphically speaking, a function is continuous at a point if the graph does not come apart at that point. We can illustrate the concept with a few graphs (Figure 1.15):

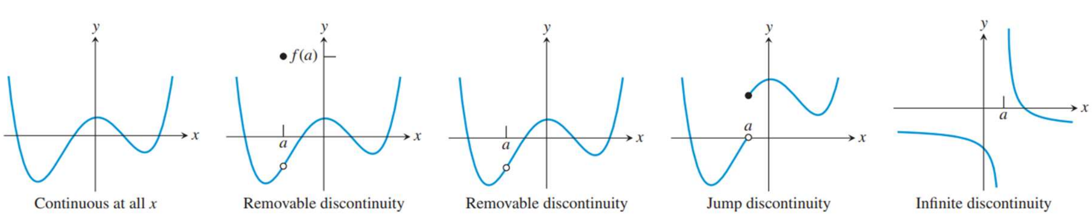


Figure 1.15 The graph of a continuous function and graphs of four other functions illustrating various types of discontinuities at $x = a$.

Increasing and Decreasing Functions

Another function concept that is easy to understand graphically is the property of being increasing, decreasing, or constant on an interval. We illustrate the concept with a few graphs (Figure 1.19):

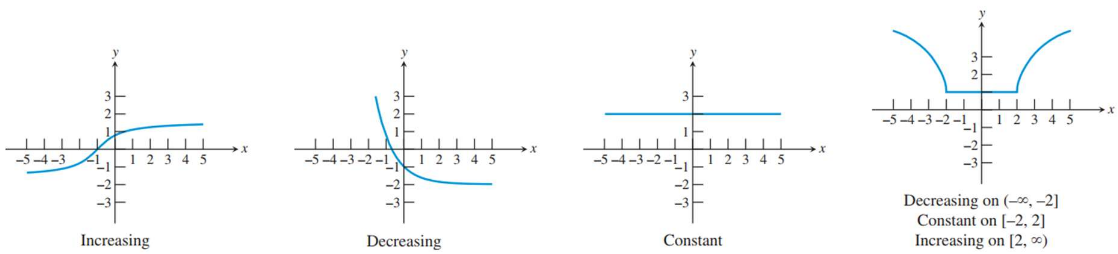


Figure 1.19 Examples of increasing, decreasing, or constant on an interval.

DEFINITION Increasing, Decreasing, and Constant Function on an Interval

A function f is **increasing** on an interval if, for any two points in the interval, a positive change in x results in a positive change in $f(x)$.

A function f is **decreasing** on an interval if, for any two points in the interval, a positive change in x results in a negative change in $f(x)$.

A function f is **constant** on an interval if, for any two points in the interval, a positive change in x results in a zero change in $f(x)$.

EXAMPLE 6 Analyzing a Function for Increasing-Decreasing Behavior

For each function, tell the intervals on which it is increasing and the intervals on which it is decreasing.

(a) $f(x) = (x + 2)^2$ **(b)** $g(x) = \dfrac{x^2}{x^2 - 1}$

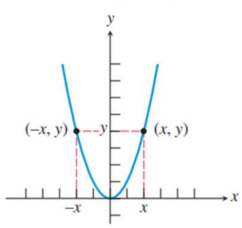
Symmetry

In the graphical sense, the word *symmetry* in mathematics carries essentially the same meaning as it does in art: The picture (in this case, the graph) "looks the same" when viewed in more than one way. The interesting thing about mathematical symmetry is that it can be characterized numerically and algebraically as well. We will be looking at three particular types of symmetry, each of which can be spotted easily from a graph, a table of values, or an algebraic formula, once you know what to look for. Since it is the connections among the three models (graphical, numerical, and algebraic) that we need to emphasize in this section, we will illustrate the various symmetries in all three ways, side-by-side.

Symmetry with respect to the *y*-axis

Example: $f(x) = x^2$

Graphically



Numerically

x	$f(x)$
−3	9
−2	4
−1	1
1	1
2	4
3	9

Algebraically

For all x in the domain of f,

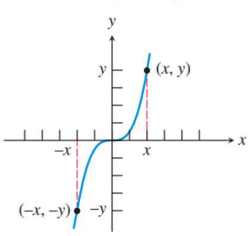
$$f(-x) = f(x).$$

Functions with this property (for example, x^n, n even) are **even** functions.

Symmetry with respect to the origin

Example: $f(x) = x^3$

Graphically



Numerically

x	y
−3	−27
−2	−8
−1	−1
1	1
2	8
3	27

Algebraically

For all x in the domain of f,

$$f(-x) = -f(x).$$

Functions with this property (for example, x^n, n odd) are **odd** functions.

What do you think about symmetry about the x-axis? is this allowed? (hint: recall function definition)

Exercises: 1, 4, 5, 9, 11, 17, 22, 29, 33, 51, 54, 57, 62, 63-66