

LESSON 3-2 SOLVING LINEAR EQUATIONS

OVERVIEW

A **linear equation** is an equation in which each term is either a constant or the product of a constant and the first power of a single variable such as $2x + 1 = 7$. To solve a linear equation, isolate the variable on one side of the equation by doing the same thing on both sides until the resulting equation has the form, *variable* = constant, as in $x = 3$.

BASIC TYPES OF SAT EQUATIONS

Some basic types of equations that appear on the SAT may look a little different than the equations encountered in your regular mathematics classes, but they can be solved using the same procedures.

➡ Example

If $\frac{r}{s} = 6$, what is the value of $\frac{4s}{r}$?

Solution

If $\frac{r}{s} = 6$, then inverting both sides gives $\frac{s}{r} = \frac{1}{6}$:

$$\frac{4s}{r} = 4\left(\frac{s}{r}\right) = 4\left(\frac{1}{6}\right) = \frac{4}{6} = \frac{2}{3}$$

Grid-in 2/3

TIP

In your regular math class, solving for x typically gives you the final answer to the problem. But x is not always the final answer on the SAT. When reading a question, circle or underline the quantity that the question asks you to find. After you solve the problem, check that your answer matches what you were required to find by looking back at what you circled or underlined.

➡ **Example**

If $kx - 19 = k - 1$ and $k = 3$, what is the value of $x + k$?

Solution

Replace k with 3 and solve the resulting equation in the usual way:

$$3x - 19 = 3 - 1$$

$$3x - 19 = 2$$

$$3x = 21$$

$$\frac{3x}{3} = \frac{21}{3}$$

$$x = 7$$

$$x + k = 7 + 3 = 10$$

Grid-in 10

➡ **Example**

If $2y - 7 = 18$, what is the value of $2y + 3$?

(A) 15

(B) 18

(C) 27

(D) 28

Solution

Rather than first solving the equation for y , solve for the quantity asked for directly. To get $2y + 3$ from $2y - 7$, add 10 to both sides of the equation:

$$2(y - 7) + 10 = 18 + 10$$

$$2y + 3 = 28$$

The correct choice is **(D)**.

➡ **Example**

When 6 times a number x is added to 5, the result is 19. What number results when 5 is subtracted from 3 times x ?

(A) -3

(B) $\frac{2}{3}$

(C) 2

(D) 7

Solution

According to the conditions of the problem, $6x + 5 = 19$, and you must find the value of $3x - 5$.

- If $6x + 5 = 19$, then $6x = 19 - 5$ so $6x = 14$.
- Dividing each side of $6x = 14$ by 2 gives $3x = 7$, and subtracting 5 from each side yields $3x - 5 = 2$.

The correct choice is (C).

➡ Example

If $\frac{5}{x} = \frac{9}{x+12}$, what is the value of $\frac{x}{3}$?

Solution

If $\frac{5}{x} = \frac{9}{x+12}$, then cross-multiplying makes $5(x + 12) = 9x$. Remove the parentheses by multiplying each term inside the parentheses by 5:

$$5x + 60 = 9x$$

$$4x = 60 \quad \leftarrow \text{To get } \frac{x}{3} \text{ from } 4x, \text{ divide both sides by 12}$$

$$\frac{4x}{12} = \frac{60}{12}$$

$$\frac{x}{3} = 5$$

Grid-in 5

➡ Example

If $\frac{13}{16}x - \frac{3}{8}x = \frac{2}{5} + \frac{3}{10}$, what is the value of x ?

(A) $\frac{17}{8}$

(B) $\frac{9}{4}$

(C) $\frac{8}{5}$

(D) $\frac{17}{10}$

Solution

Combine fractions on each side of the equation:

$$\begin{aligned}\frac{13}{16}x - \frac{3}{8}x &= \frac{2}{5} + \frac{3}{10} \\ \frac{13}{16}x - \frac{6}{16}x &= \frac{4}{10} + \frac{3}{10} \\ \frac{7}{16}x &= \frac{7}{10} \quad \leftarrow \text{Isolate } x \text{ by multiplying both sides} \\ &\quad \text{by the reciprocal of its coefficient}\end{aligned}$$

$$\begin{aligned}\frac{16}{7} \left(\frac{7}{16}x \right) &= \frac{16}{7} \left(\frac{7}{10} \right) \\ 1 \cdot x &= \frac{16}{10} \\ x &= \frac{8}{5}\end{aligned}$$

The correct choice is **(C)**.

Solving a Linear Equation Using Two Operations

To isolate a variable in a linear equation, it may be necessary to add or subtract and then to multiply or divide. If an equation contains parentheses, remove them by multiplying each term inside the parentheses by the term outside the parentheses.

➡ Example

Solve for b :

$$3(b + 2) + 2b = 21$$

Solution

- Remove the parentheses by multiplying each term inside the parentheses by 3:
- Combine like terms:
- Subtract 6 from each side of the equation:
- Simplify:
- Divide each side of the equation by 5:

$$3(b + 2) + 2b = 21$$

$$3b + 6 + 2b = 21$$

$$5b + 6 = 21$$

$$5b + 6 - 6 = 21 - 6$$

$$5b = 15$$

$$\frac{5b}{5} = \frac{15}{5}$$

$$b = 3$$

➡ Example

$$\frac{7}{3}\left(x + \frac{9}{28}\right) - 3 = 17$$

Which value of x satisfies the equation above?

(A) $\frac{33}{4}$

(B) $\frac{249}{28}$

(C) $\frac{77}{4}$

(D) $\frac{45}{28}$

Solution

$$\begin{aligned}
 \frac{7}{3}\left(x + \frac{9}{28}\right) &= 20 \\
 \frac{7}{3}x + \left(\frac{\cancel{7}}{\cancel{3}}\right)\left(\frac{\cancel{9}^3}{\cancel{28}_4}\right) &= 20 \\
 \frac{7}{3}x &= 20 - \frac{3}{4} \\
 \frac{7}{3}x &= \frac{80}{4} - \frac{3}{4} \\
 \frac{3}{7} \cdot \left(\frac{7}{3}x\right) &= \frac{3}{\cancel{7}} \cdot \left(\frac{\cancel{7}^{11}}{4}\right) \\
 x &= \frac{33}{4}
 \end{aligned}$$

The correct choice is **(A)**.

➡ Example

$$\frac{2(m+4)+13}{5} = \frac{21-(8-3m)}{4}$$

In the equation above, what is the value of m ?

Solution

- Simplify the numerators:

$$\frac{2m+21}{5} = \frac{13+3m}{4}$$

- Cross-multiply and simplify:

$$\begin{aligned}
 5(13+3m) &= 4(2m+21) \\
 65+15m &= 8m+84
 \end{aligned}$$

- Isolate the variable:

$$\begin{aligned}
 15m-8m &= 84-65 \\
 7m &= 19 \\
 m &= \frac{19}{7}
 \end{aligned}$$