1-1 Points, Lines, and Planes

GET READY for the Lesson

Have you ever noticed that a four-legged chair sometimes wobbles, but a three-legged stool never wobbles? This is an example of points and how they lie in a plane. All geometric shapes are made of points. In this book, you will learn about those shapes and their characteristics.

Main Ideas

- Identify and model points, lines, and planes.
- Identify collinear and coplanar points and intersecting lines and planes in space.

Standard 8.0 **Students demonstrate understanding by identifying and giving examples of undefined terms,** axioms, theorems, and inductive and deductive reasoning. (Key)

New Vocabulary

undefined term
point
line
collinear
plane
coplanar
space
locus

Reading Math

Noncollinear and Noncoplanar

The word *noncollinear* means not collinear or not lying on the same line. Likewise, *noncoplanar* means not lying in the same plane.

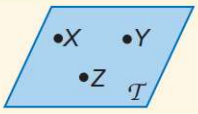
Name Points, Lines, and Planes You are familiar with the terms *plane*, *line*, and *point* from algebra. You graph on a coordinate *plane*, and ordered pairs represent *points* on *lines*. In geometry, these terms have similar meanings.

Unlike objects in the real world that model these shapes, points, lines, and planes do not have any actual size. In geometry, *point*, *line*, and *plane* are considered **undefined terms** because they are only explained using examples and descriptions.

- A **point** is simply a location.
- A **line** is made up of points and has no thickness or width. Points on the same line are said to be **collinear**.
- A **plane** is a flat surface made up of points. Points that lie on the same plane are said to be **coplanar**. A plane has no depth and extends infinitely in all directions.

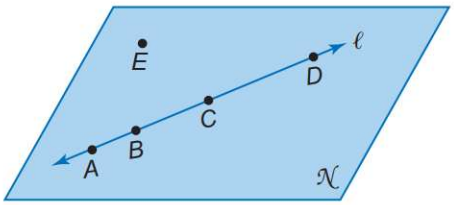
Points are often used to name lines and planes.

KEY CONCEPT — *Points, Lines, and Planes*

	Point	Line	Plane
Model			
Drawn	as a dot	with an arrowhead at each end	as a shaded, slanted 4-sided figure
Named by	a capital letter	the letters representing two points on the line or a lowercase script letter	a capital script letter or by the letters naming three noncollinear points
Facts	A point has neither shape nor size.	There is exactly one line through any two points.	There is exactly one plane through any three noncollinear points.
Words/ Symbols	point P	line n, line AB or $\overleftrightarrow{AB}$, line BA or $\overleftrightarrow{BA}$	plane $\mathcal{T}$, plane XYZ, plane XZY, plane YXZ, plane YZX, plane ZXY, plane ZYX

EXAMPLE 1 Name Lines and Planes

Use the figure to name each of the following.



a. a line containing point *A*

The line can be named as line ℓ. There are four points on the line. Any two of the points can be used to name the line.

$\overleftrightarrow{AB}$ $\overleftrightarrow{BA}$ $\overleftrightarrow{AC}$ $\overleftrightarrow{CA}$ $\overleftrightarrow{AD}$ $\overleftrightarrow{DA}$ $\overleftrightarrow{BC}$ $\overleftrightarrow{CB}$ $\overleftrightarrow{BD}$ $\overleftrightarrow{DB}$ $\overleftrightarrow{CD}$ $\overleftrightarrow{DC}$

b. a plane containing point *C*

The plane can be named as plane $\mathcal{N}$. You can also use the letters of any three *noncollinear* points to name the plane.

plane *ABE* plane *ACE* plane *ADE* plane *BCE* plane *BDE* plane *CDE*

The letters of each name can be reordered to create other names for this plane. For example, *ABE* can be written as *AEB*, *BEA*, *BAE*, *EBA*, and *EAB*.

Study Tip

Dimension

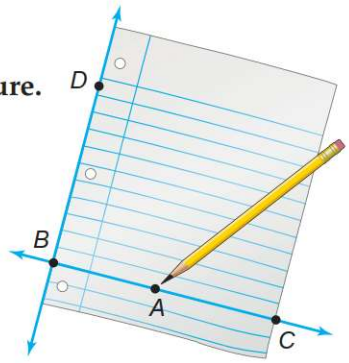
A point has no dimension. A line exists in one dimension. However, a square is two-dimensional, and a cube is three-dimensional.

CHECK Your Progress

1. Use the figure to name a plane containing points *A* and *D*.

EXAMPLE 2 Model Points, Lines, and Planes

Name the geometric shapes modeled by the picture.



The pencil point models point *A*.

The blue rule on the paper models line *BC*.

The edge of the paper models line *BD*.

The sheet of paper models plane *ADC*.

CHECK Your Progress

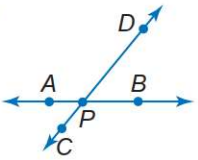
2. Name the geometric shape modeled by stripes on a sweater.

Study Tip

Naming Points

Recall that points on the coordinate plane are named using *rectangular coordinates* or *ordered pairs*. Point *G* can be named as *G*(−1, −3).

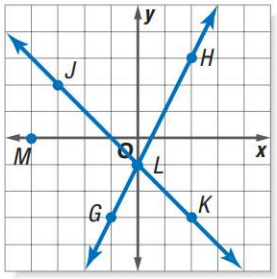
Two lines intersect in a point. In the figure at the right, point *P* represents the intersection of $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$. Lines can intersect planes, and planes can intersect each other.



EXAMPLE 3 Draw Geometric Figures

Draw and label a figure for each relationship.

a. ALGEBRA Lines *GH* and *JK* intersect at *L* for *G*(−1, −3), *H*(2, 3), *J*(−3, 2), and *K*(2, −3) on a coordinate plane. Point *M* is coplanar with these points, but not collinear with $\overleftrightarrow{GH}$ or $\overleftrightarrow{JK}$.



Graph each point and draw $\overleftrightarrow{GH}$ and $\overleftrightarrow{JK}$.

Label the intersection point as *L*.

An infinite number of points are coplanar with *G*, *H*, *J*, *K*, and *L*, but not collinear with $\overleftrightarrow{GH}$ or $\overleftrightarrow{JK}$. In the graph, one such point is *M*(−4, 0).

Study Tip

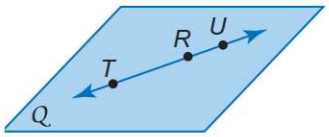
Three-Dimensional Drawings

Because it is impossible to show space or an entire plane in a figure, edged shapes with different shades of color are used to represent planes. If the lines are hidden from view, the lines or segments are shown as dashed lines or segments.

b. $\overleftrightarrow{TU}$ lies in plane Q and contains point R.

Draw a surface to represent plane Q and label it.
Draw a line anywhere on the plane.
Draw dots on the line for points T and U.
Since $\overleftrightarrow{TU}$ contains R, point R lies on $\overleftrightarrow{TU}$.
Draw a dot on $\overleftrightarrow{TU}$ and label it R.

The locations of points T, R, and U are totally arbitrary.



CHECK Your Progress

3. Draw and label a figure in which points A, B, and C are coplanar and B and C are collinear.

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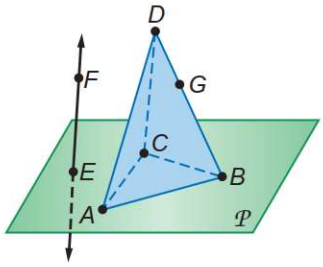
Points, Lines, and Planes in Space

Space is a boundless, three-dimensional set of all points. Space contains lines and planes.

EXAMPLE **Interpret Drawings**



4 a. How many planes appear in this figure?

There are four planes: plane $\mathcal{P}$, plane ADB, plane BCD, plane ACD.

b. Name three points that are collinear.

Points D, B, and G are collinear.

c. Are points G, A, B, and E coplanar? Explain.

Points A, B, and E lie in plane $\mathcal{P}$, but point G does not lie in plane $\mathcal{P}$. Thus, they are not coplanar. Points A, G, and B lie in a plane, but point E does not lie in plane AGB.

d. At what point do $\overleftrightarrow{EF}$ and $\overleftrightarrow{AB}$ intersect?

$\overleftrightarrow{EF}$ and $\overleftrightarrow{AB}$ do not intersect. $\overleftrightarrow{AB}$ lies in plane $\mathcal{P}$, but only point E lies in $\mathcal{P}$.

CHECK Your Progress

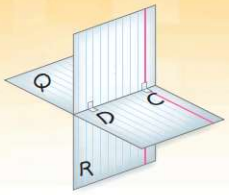
4. Name the intersection of plane BCD and plane $\mathcal{P}$.

GEOMETRY LAB

Modeling Intersecting Planes

- Label one index card as Q and another as R.
- Hold the two index cards together and cut a slit halfway through both cards.

- Hold the cards so that the slits meet and insert one card into the slit of the other. Use tape to hold the cards together.
- Where the two cards meet models a line. Draw the line and label two points, C and D, on the line.



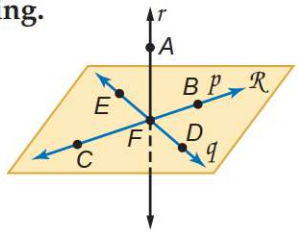
ANALYZE

1. Draw a point F on your model so that it lies in Q but not in R. Can F lie on $\overleftrightarrow{DC}$? Explain.
2. If point H lies in both Q and R, where would it lie? Draw point H on your model.
3. Draw a sketch of your model on paper. Label all points, lines, and planes.

CHECK Your Understanding

Example 1 (p. 7)

Use the figure at the right to name each of the following.



1. a line containing point B
2. a plane containing points D and C

Example 2 (p. 7)

Name the geometric term modeled by each object.

3. the beam from a laser
4. a ceiling

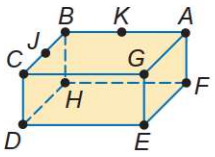
Example 3 (pp. 7–8)

Draw and label a figure for each relationship.

5. A line in a coordinate plane contains $X(3, -1)$, $Y(-3, -4)$, and $Z(-1, -3)$ and a point W that does not lie on $\overleftrightarrow{XY}$.
6. Plane Q contains lines r and s that intersect in P.

Example 4 (p. 8)

For Exercises 7–9, refer to the figure.



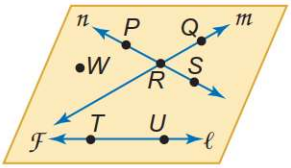
7. How many planes are shown in the figure?
8. Name three points that are collinear.
9. Are points A, C, D, and J coplanar? Explain.

Exercises

HOMEWORK HELP

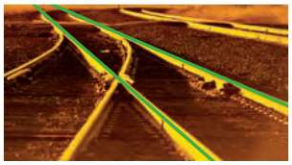
For Exercises	See Examples
10–15	1
16–22	2
23–30	3
31–34	4

Refer to the figure.



10. Name a line that contains point P.
11. Name the plane containing lines n and m.
12. Name the intersection of lines n and m.
13. Name a point not contained in lines ℓ, m, or n.
14. What is another name for line n?
15. Does line ℓ intersect line m or line n? Explain.

Name the geometric term(s) modeled by each object.

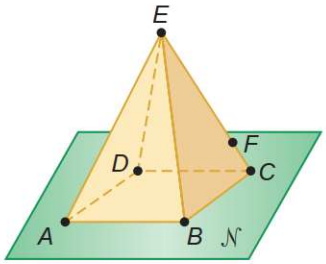
16. 
17. 
18. 

19. a tablecloth
20. a partially-opened newspaper
21. woven threads in a piece of cloth
22. a knot in a string

Draw and label a figure for each relationship.

23. Line AB intersects plane Q at W.
24. Point T lies on $\overleftrightarrow{WR}$.
25. Points $Z(4, 2)$, $R(-4, 2)$, and S are collinear, but points Q, Z, R, and S are not.
26. The coordinates for points C and R are $(-1, 4)$ and $(6, 4)$, respectively. $\overleftrightarrow{RS}$ and $\overleftrightarrow{CD}$ intersect at $P(3, 2)$.

Refer to the figure.



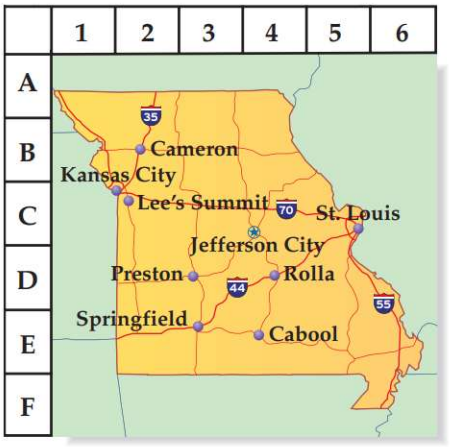
27. How many planes are shown in the figure?

28. How many planes contain points B, C, and E?

29. Name three collinear points.

30. Where could you add point G on plane N so that A, B, and G would be collinear?

MAPS For Exercises 31–34, refer to the map, and use the following information. A map represents a plane. Locations on this plane are named using a letter/number combination.



31. Name the letter/number combination where St. Louis is located.

32. Name the letter/number combination where Springfield is located.

33. What city is located at (B, 2)?

34. What city is located at (D, 4)?

Real-World Career
Engineering Technician
Engineering technicians or drafters use perspective to create drawings used in construction, and manufacturing. Technicians must have knowledge of math, science, and engineering.

MathOnline
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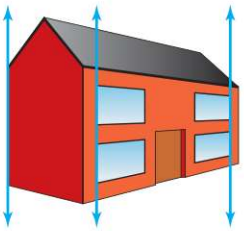
ONE-POINT PERSPECTIVE One-point perspective drawings use lines to convey depth in a picture. Lines representing horizontal lines in the real object can be extended to meet at a single point called the *vanishing point*.



35. Trace the figure at the right. Draw all of the vertical lines. Three are already drawn for you.

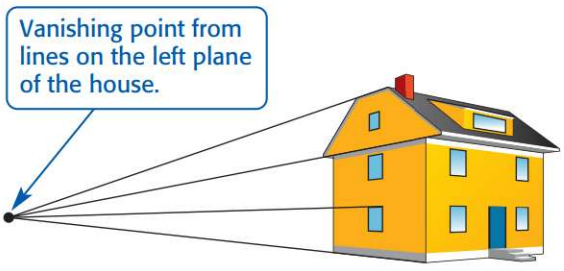
36. Draw and extend the horizontal lines to locate the vanishing point and label it.

37. Draw a one-point perspective of your classroom or a room in your house.

38. RESEARCH Use the Internet or other research resources to investigate one-point perspective drawings in which the vanishing point is in the center of the picture. How do they differ from the drawing for Exercises 35–37?

TWO-POINT PERSPECTIVE Two-point perspective drawings also use lines to convey depth, but two sets of lines can be drawn to meet at two vanishing points.

39. Trace the outline of the house. Draw all of the vertical lines.



40. Draw and extend the lines on your sketch representing horizontal lines in the real house to identify the vanishing point on the right plane in this figure.

41. Which type of lines seems to be unaffected by any type of perspective drawing?

Michael Newman/PhotoEdit

EXTRA PRACTICE
See pages 800, 828.
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Another way to describe a group of points is called a locus. A locus is a set of points that satisfy a particular condition.

42. Find five points that satisfy the equation $4 - x = y$. Graph them on a coordinate plane and describe the geometric figure they suggest.

43. Find ten points that satisfy the inequality $y > -2x + 1$. Graph them on a coordinate plane and describe the geometric figure they suggest.

H.O.T. Problems

44. OPEN ENDED Fold a sheet of paper. Open the paper and fold it again in a different way. Open the paper and label the geometric figures you observe. Describe the figures.

45. FIND THE ERROR Raymond and Micha were looking for patterns to determine how many ways there are to name a plane given a certain number of points. Who is correct? Explain your reasoning.

Raymond	Micha
If there are 4 points, then there are $4 \cdot 3 \cdot 2$ ways to name the plane.	If there are 5 noncollinear points, then there are $5 \cdot 4 \cdot 3$ ways to name the plane.

46. CHALLENGE Describe a real-life example of three lines in space that do not intersect each other and no two of which lie in the same plane.

47. *Writing in Math* Refer to the information about chairs on page 6. Explain how the chair legs relate to points in a plane. Include how many legs would create a chair that does not wobble.

STANDARDS PRACTICE

48. Four lines are coplanar. What is the *greatest* number of intersection points that can exist?

A 4

B 5

C 6

D 7

49. REVIEW What is the value of x if $-5x + 4 = -6$?

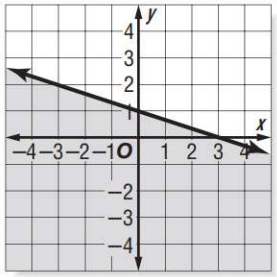
F -5

G -2

H 2

J 5

50. REVIEW Which inequality is shown on the graph below?



A $y > -\frac{1}{3}x + 1$

B $y < -\frac{1}{3}x + 1$

C $y \leq -\frac{1}{3}x + 1$

D $y \geq -\frac{1}{3}x + 1$

GET READY for the Next Lesson

PREREQUISITE SKILL **Replace each ● with >, <, or = to make a true statement.**

51. $\frac{1}{2}$ in. ● $\frac{3}{8}$ in.

52. $\frac{4}{16}$ in. ● $\frac{1}{4}$ in.

53. $\frac{4}{5}$ in. ● $\frac{6}{10}$ in.

54. 10 mm ● 1 cm

55. 2.5 cm ● 28 mm

56. 0.025 cm ● 25 mm

READING MATH

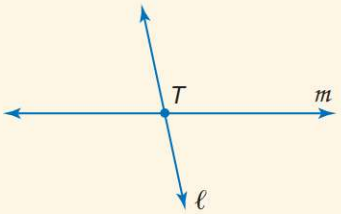
Standard 1.0 Students demonstrate understanding by identifying and giving examples of undefined terms, axioms, theorems, and inductive and deductive reasoning. (Key)

Describing What You See

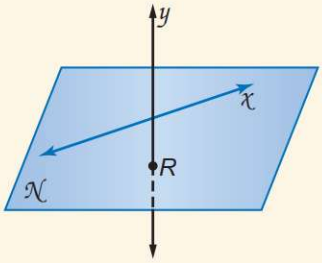
Figures play an important role in understanding geometric concepts. It is helpful to know what words and phrases can be used to describe figures. Likewise, it is important to know how to read a geometric description and be able to draw the figure it describes.

The figures and descriptions below help you visualize and write about points, lines, and planes.

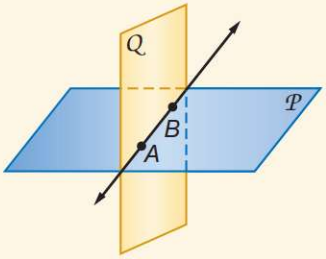
Point P is on line m.
Line m contains P.
Line m passes through P.



Lines ℓ and m intersect in T.
Point T is the interesection of lines ℓ and m.
Point T is on line m. Point T is on line ℓ.



Line x and point R are in $\mathcal{N}$.
Point R lies in $\mathcal{N}$.
Plane $\mathcal{N}$ contains R and line x.
Line y intersects $\mathcal{N}$ at R.
Point R is the intersection of line y with $\mathcal{N}$.
Lines y and x do not intersect.

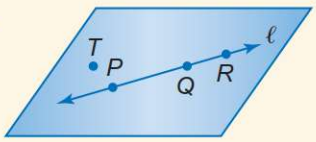


$\overleftrightarrow{AB}$ is in $\mathcal{P}$ and Q.
Points A and B lie in both $\mathcal{P}$ and Q.
Planes $\mathcal{P}$ and Q both contain $\overleftrightarrow{AB}$.
Planes $\mathcal{P}$ and Q intersect in $\overleftrightarrow{AB}$.
$\overleftrightarrow{AB}$ is the intersection of $\mathcal{P}$ and Q.

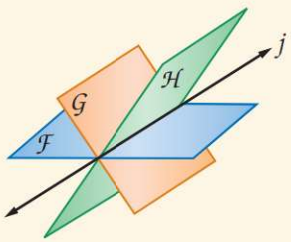
Reading to Learn

Write a description for each figure.

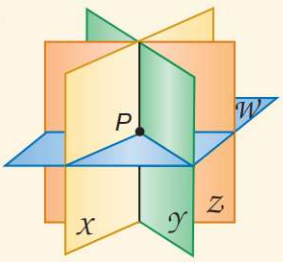
1.



2.



3.



4. Draw and label a figure for the statement *Planes $\mathcal{A}$, $\mathcal{B}$, and C do not intersect.*