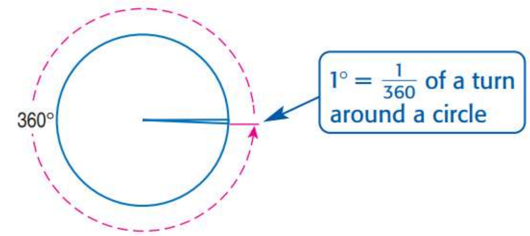
# 1-4 Angle Measure

## GET READY for the Lesson

Astronomer Claudius Ptolemy based his observations of the solar system on a unit that resulted from dividing the circumference, or the distance around, a circle into 360 parts. This later became known as a **degree**. In this lesson, you will learn to measure angles in degrees.




### Main Ideas

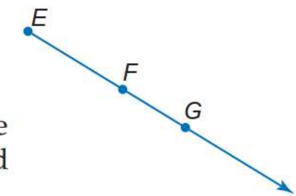
- Measure and classify angles.
- Identify and use congruent angles and the bisector of an angle.

**Standard 16.0 Students perform basic constructions with a straightedge and compass, such as angle bisectors,** perpendicular bisectors, and the line parallel to a given line through a point off the line. (Key)

### New Vocabulary

degree
ray
opposite rays
angle
sides
vertex
interior
exterior
right angle
acute angle
obtuse angle
angle bisector

**Measure Angles** A **ray** is part of a line. It has one endpoint and extends indefinitely in one direction. Rays are named stating the endpoint first and then any other point on the ray. The figure at the right shows ray *EF*, which can be symbolized as $\overrightarrow{EF}$. This ray could also be named as $\overrightarrow{EG}$, but not as $\overrightarrow{FE}$ because *F* is not the endpoint of the ray.




If you choose a point on a line, that point determines exactly two rays called **opposite rays**. Line *m*, shown below, is separated into two opposite rays, $\overrightarrow{PQ}$ and $\overrightarrow{PR}$. Point *P* is the common endpoint of those rays. $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ are collinear rays.

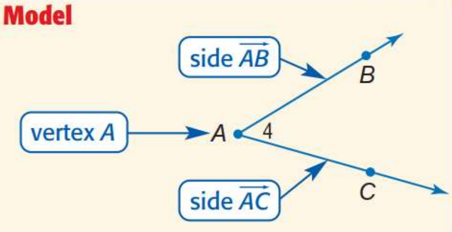

An **angle** is formed by two *noncollinear* rays that have a common endpoint. The rays are called **sides** of the angle. The common endpoint is the **vertex**.

**KEY CONCEPT** — **Angle**

**Words** An angle is formed by two noncollinear rays that have a common endpoint.

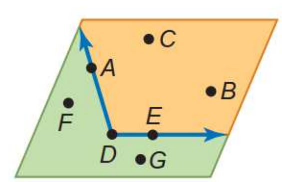
**Symbols** $\angle A$, $\angle BAC$, $\angle CAB$, $\angle 4$

**Model**




An angle divides a plane into three distinct parts.

- Points *A*, *D*, and *E* lie on the angle.
- Points *C* and *B* lie in the **interior** of the angle.
- Points *F* and *G* lie in the **exterior** of the angle.

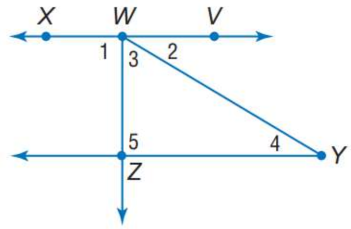


**Study Tip**

**Naming Angles**

You can name an angle by a single letter *only* when there is one angle shown at that vertex.

## EXAMPLE 1 Angles and Their Parts




**a. Name all angles that have $W$ as a vertex.**

$\angle 1, \angle 2, \angle 3, \angle XWY, \angle ZWV, \angle YWV$

**b. Name the sides of ∠1.**

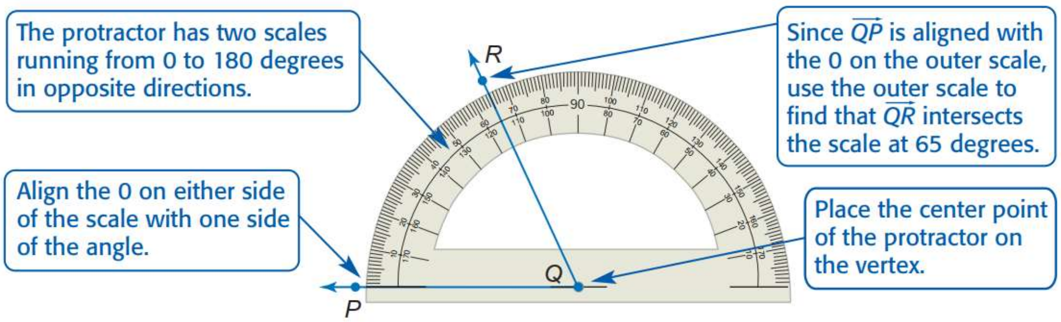
$\overrightarrow{WZ}$ and $\overrightarrow{WX}$ are the sides of $\angle 1$.

**c. Write another name for ∠WYZ.**

$\angle 4$, $\angle Y$, and $\angle ZYW$ are other names for $\angle WYZ$.

**CHECK Your Progress** **1.** Name a pair of opposite rays.

To measure an angle, you can use a *protractor*. Angle $PQR$ is a 65 degree (65°) angle. We say that the *degree measure* of $\angle PQR$ is 65, or simply $m\angle PQR = 65$.

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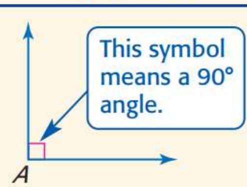
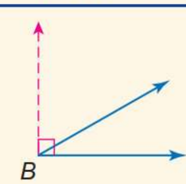
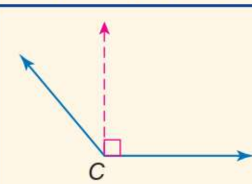
Angles can be classified by their measures.

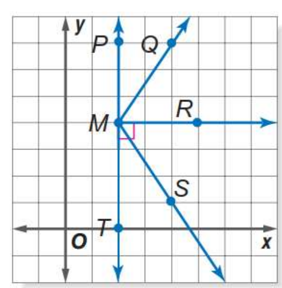
**Reading Math**

**Angles** Opposite rays are also known as a *straight angle*. Its measure is 180°. Unless otherwise specified in this book, the term *angle* means a nonstraight angle.

**KEY CONCEPT** *Classify Angles*

| Name | right angle | acute angle | obtuse angle |
|---|---|---|---|
| **Measure** | $m\angle A = 90$ | $m\angle B < 90$ | $180 > m\angle C > 90$ |
| **Model** |   |   |   |

## EXAMPLE 2 Measure and Classify Angles




**Measure each angle and classify as *right, acute,* or *obtuse*.**

**a.** $\angle PMQ$

Use a protractor to find that $m\angle PMQ = 30$.
$30 < 90$, so $\angle PMQ$ is an acute angle.

**b.** $\angle TMR$

$\angle TMR$ is marked with a right angle symbol, so measuring is not necessary; $m\angle TMR = 90$.

**CHECK Your Progress**

**2.** Measure $\angle QMT$ and classify it as *right*, *acute*, or *obtuse*.

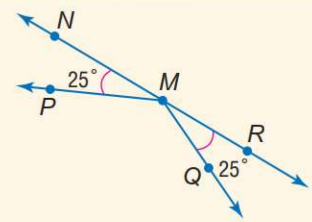
**Congruent Angles** Just as segments that have the same measure are congruent, angles that have the same measure are congruent.

**KEY CONCEPT** — *Congruent Angles*

**Words** Angles that have the same measure are congruent angles. Arcs on the figure indicate which angles are congruent.

**Symbols** $\angle NMP \cong \angle QMR$

**Model**




You can construct an angle congruent to a given angle without knowing the measure of the angle.

## CONSTRUCTION

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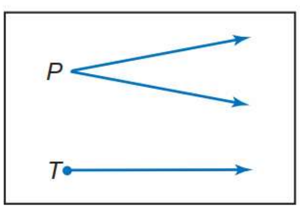
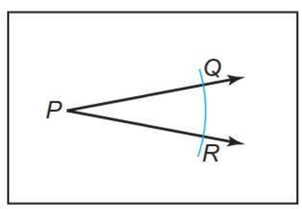
### Copy an Angle

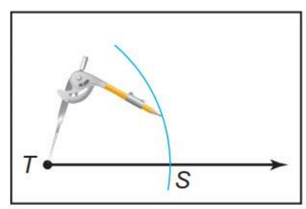
**Step 1**
Draw an angle like $\angle P$ on your paper. Use a straightedge to draw a ray on your paper. Label its endpoint $T$.



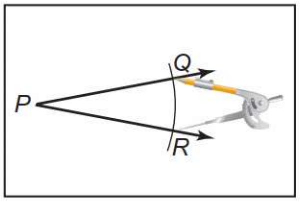

**Step 2**
Place the tip of the compass at point $P$ and draw a large arc that intersects both sides of $\angle P$. Label the points of intersection $Q$ and $R$.



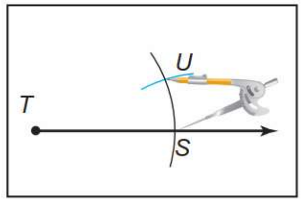

**Step 3**
Using the same compass setting, put the compass at $T$ and draw a large arc that intersects the ray. Label the point of intersection $S$.



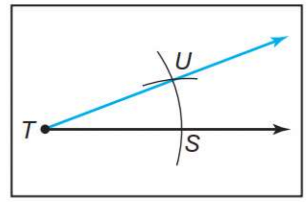

**Step 4**
Place the point of your compass on $R$ and adjust so that the pencil tip is on $Q$.




**Step 5**
Without changing the setting, place the compass at $S$ and draw an arc to intersect the larger arc you drew in Step 3. Label the point of intersection $U$.

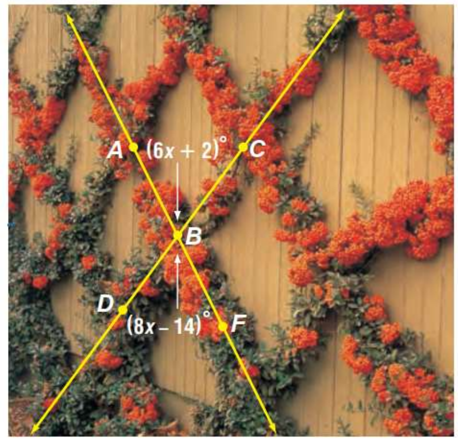



**Step 6**
Use a straightedge to draw $\overrightarrow{TU}$.



## EXAMPLE Use Algebra to Find Angle Measures

**3** **GARDENING** **A trellis is often used to provide a frame for vining plants. Some of the angles formed by the slats of the trellis are congruent angles. In the figure, $\angle ABC \cong \angle DBF$. If $m\angle ABC = 6x + 2$ and $m\angle DBF = 8x - 14$, find the actual measurements of $\angle ABC$ and $\angle DBF$.**




| | |
|---|---|
| $\angle ABC \cong \angle DBF$ | Given |
| $m\angle ABC = m\angle DBF$ | Definition of congruent angles |
| $6x + 2 = 8x - 14$ | Substitution |
| $6x + 16 = 8x$ | Add 14 to each side. |
| $16 = 2x$ | Subtract 6*x* from each side. |
| $8 = x$ | Divide each side by 2. |

Use the value of $x$ to find the measure of one angle.

| | |
|---|---|
| $m\angle ABC = 6x + 2$ | Given |
| $= 6(8) + 2$ | $x = 8$ |
| $= 48 + 2$ or $50$ | Simplify. |

Since $m\angle ABC = m\angle DBF$, $m\angle DBF = 50$. Both $\angle ABC$ and $\angle DBF$ measure 50.

### Study Tip

**Checking Solutions**

Check that you have computed the value of *x* correctly by substituting the value into the expression for $\angle DBF$. If you don't get the same measure as $\angle ABC$, you have made an error.

### CHECK Your Progress

**3.** Suppose $\angle JKL \cong \angle MKN$. If $m\angle JKL = 5x + 4$ and $m\angle MKN = 3x + 12$, find the actual measurements of the two angles.

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## GEOMETRY LAB

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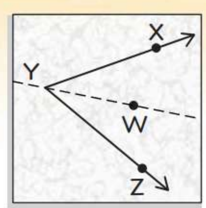
### Bisect an Angle

**MAKE A MODEL**

- Draw any $\angle XYZ$ on patty paper or tracing paper.
- Fold the paper through point *Y* so that $\overrightarrow{YX}$ and $\overrightarrow{YZ}$ are aligned together.
- Open the paper and label a point on the crease in the interior of $\angle XYZ$ as point *W*.



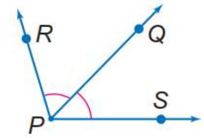

**ANALYZE THE MODEL**

**1.** What seems to be true about $\angle XYW$ and $\angle WYZ$?

**2.** Measure $\angle XYZ$, $\angle XYW$, and $\angle WYZ$.

**3.** You learned about segment bisectors in Lesson 1-3. **Make a conjecture** about the term *angle bisector*.

A ray that divides an angle into two congruent angles is called an **angle bisector**. If $\overrightarrow{PQ}$ is the angle bisector of $\angle RPS$, then point $Q$ lies in the interior of $\angle RPS$, and $\angle RPQ \cong \angle QPS$. A line segment can also bisect an angle.



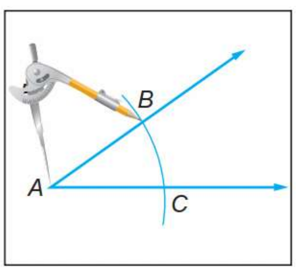

Just as with segments, when a line divides an angle into smaller angles, the sum of the measures of the smaller angles equals the measure of the largest angle. So in the figure, $m\angle RPS = m\angle RPQ + m\angle QPS$.

You can construct the angle bisector of any angle without knowing the measure of the angle.

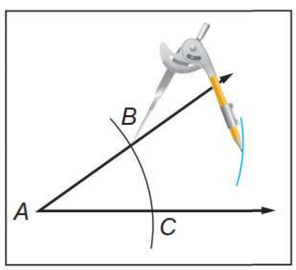
## CONSTRUCTION

### Bisect an Angle

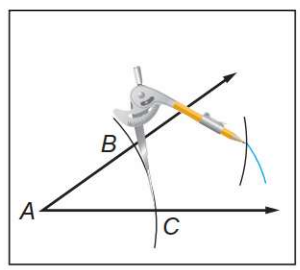
**Step 1**
Draw an angle and label the vertex as $A$. Put your compass at point $A$ and draw a large arc that intersects both sides of $\angle A$. Label the points of intersection $B$ and $C$.



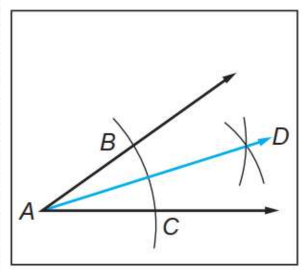

**Step 2**
With the compass at point $B$, draw an arc in the interior of the angle.




**Step 3**
Keeping the same compass setting, place the compass at point $C$ and draw an arc that intersects the arc drawn in Step 2.




**Step 4**
Label the point of intersection $D$. Draw $\overrightarrow{AD}$. $\overrightarrow{AD}$ is the bisector of $\angle A$. Thus, $m\angle BAD = m\angle DAC$ and $\angle BAD \cong \angle DAC$.




## CHECK Your Understanding

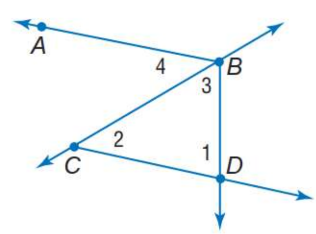
**Example 1** (p. 32)

**For Exercises 1–3, use the figure at the right.**

**1.** Name the vertex of $\angle 2$.

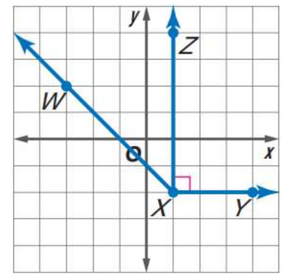
**2.** Name the sides of $\angle 4$.

**3.** Write another name for $\angle BDC$.



**Example 2** (p. 32)

**Measure each angle and classify as *right*, *acute*, or *obtuse*.**

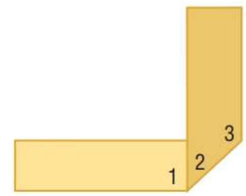



**4.** $\angle WXY$

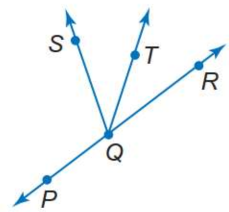
**5.** $\angle WXZ$

**Example 3** (p. 34)

**6. ORIGAMI** The art of origami involves folding paper at different angles to create designs and three-dimensional figures. One of the folds in origami involves folding a strip of paper so that the lower edge of the strip forms a right angle with itself. Identify each numbered angle as *right*, *acute*, or *obtuse*.




**ALGEBRA In the figure, $\overrightarrow{QP}$ and $\overrightarrow{QR}$ are opposite rays, and $\overrightarrow{QT}$ bisects $\angle RQS$.**




**7.** If $m\angle RQT = 6x + 5$ and $m\angle SQT = 7x - 2$, find $m\angle RQT$.

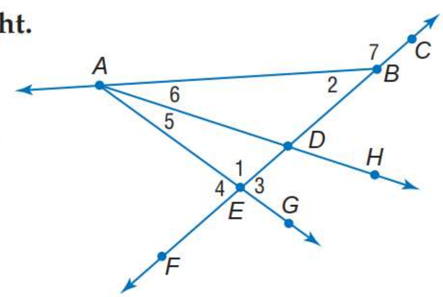
**8.** Find $m\angle TQS$ if $m\angle RQS = 22a - 11$ and $m\angle RQT = 12a - 8$.

## Exercises

**HOMEWORK HELP**

| For Exercises | See Examples |
|---|---|
| 9–24 | 1 |
| 25–30 | 2 |
| 31–36 | 3 |

**For Exercises 9–24, use the figure on the right. Name the vertex of each angle.**




**9.** $\angle 1$ **10.** $\angle 2$

**11.** $\angle 6$ **12.** $\angle 5$

**Name the sides of each angle.**

**13.** $\angle ADB$ **14.** $\angle 6$

**15.** $\angle 3$ **16.** $\angle 5$

**Write another name for each angle.**

**17.** $\angle 7$ **18.** $\angle AEF$ **19.** $\angle ABD$ **20.** $\angle 1$

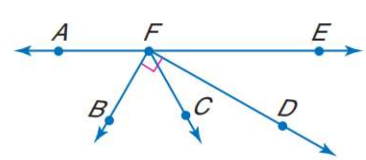
**21.** Name a point in the interior of $\angle GAB$.

**22.** Name an angle with vertex $B$ that appears to be acute.

**23.** Name a pair of angles that share exactly one point.

**24.** Name a point in the interior of $\angle CEG$.

**Measure each angle and classify as *right*, *acute*, or *obtuse*.**

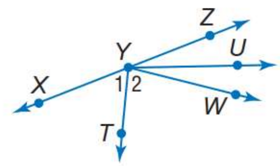



**25.** $\angle BFD$ **26.** $\angle AFB$

**27.** $\angle DFE$ **28.** $\angle EFC$

**29.** $\angle AFD$ **30.** $\angle EFB$

**ALGEBRA In the figure, $\overrightarrow{YX}$ and $\overrightarrow{YZ}$ are opposite rays. $\overrightarrow{YU}$ bisects $\angle ZYW$, and $\overrightarrow{YT}$ bisects $\angle XYW$.**




**31.** If $m\angle ZYU = 8p - 10$ and $m\angle UYW = 10p - 20$, find $m\angle ZYU$.

**32.** If $m\angle 1 = 5x + 10$ and $m\angle 2 = 8x - 23$, find $m\angle 2$.

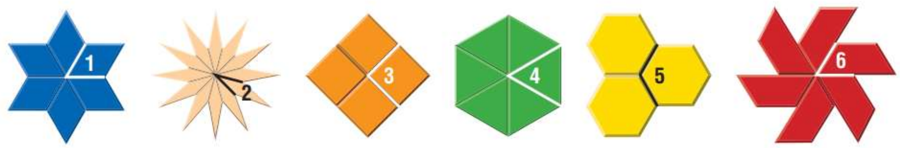
**33.** If $m\angle 1 = y$ and $m\angle XYW = 6y - 24$, find $y$.

**34.** If $m\angle WYZ = 82$ and $m\angle ZYU = 4r + 25$, find $r$.

**35.** If $m\angle WYX = 2(12b + 7)$ and $m\angle ZYU = 9b - 1$, find $m\angle UYW$.

**36.** If $\angle ZYW$ is a right angle and $m\angle ZYU = 13a - 7$, find $a$.

**37. PATTERN BLOCKS** Pattern blocks can be arranged to fit in a circular pattern without leaving spaces. Remember that the measurement around a full circle is 360°. Determine the degree measure of the numbered angles shown below.




**EXTRA PRACTICE**
See pages 801, 828.
**Math Online**
Self-Check Quiz at
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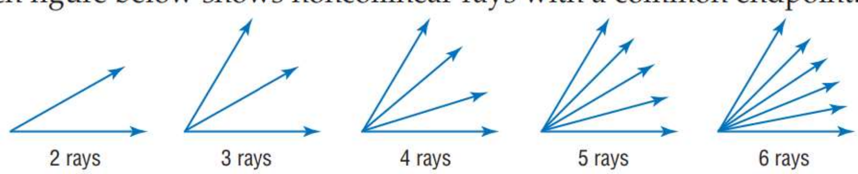
**38. RESEARCH** The words *obtuse* and *acute* have other meanings in the English language. Look these words up in a dictionary and write how the everyday meaning relates to the mathematical meaning.

**H.O.T. Problems**

**39. OPEN ENDED** Draw and label a figure to show $\overrightarrow{PR}$ that bisects $\angle SPQ$ and $\overrightarrow{PT}$ that bisects $\angle SPR$. Use a protractor to measure each angle.

**40. REASONING** Are all right angles congruent? What information would you use to support your answer?

**CHALLENGE For Exercises 41–44, use the following information.**
Each figure below shows noncollinear rays with a common endpoint.



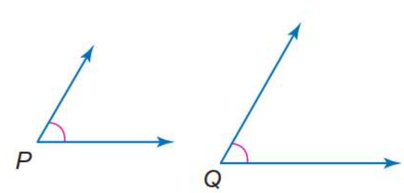

**41.** Count the number of angles in each figure.

**42.** Describe the pattern between the number of rays and the number of angles.

**43.** **Make a conjecture** about the number of angles that are formed by 7 noncollinear rays and by 10 noncollinear rays.

**44.** Write a formula for the number of angles formed by $n$ noncollinear rays with a common endpoint.

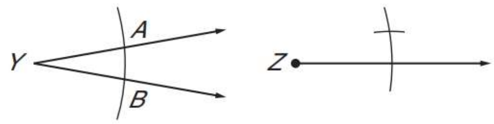
**45. REASONING** How would you compare the sizes of $\angle P$ and $\angle Q$? Explain.




**46.** ***Writing in Math*** Refer to page 31. Describe the size of a degree. Include how to find degree measure with a protractor.

## STANDARDS PRACTICE

**47.** Dominic is using a straightedge and compass to do the construction shown below.




Which *best* describes the construction Dominic is doing?

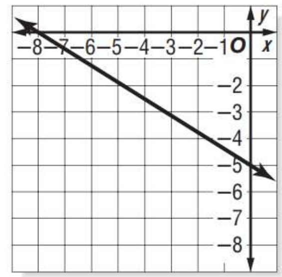
**A** a line through $Z$ that bisects $\angle AYB$

**B** a line through $Z$ parallel to $\overrightarrow{YA}$

**C** a ray through $Z$ congruent to $\overrightarrow{YA}$

**D** an angle $Z$ congruent to $\angle AYB$

**48. REVIEW** Which coordinate points represent the $x$- and $y$-intercepts of the graph below?




**F** (–5, –8), (0, 0)

**G** (0, –8), (–5, 0)

**H** (–8, 0), (0, –5)

**J** (0, –5), (0, –8)

## Spiral Review

**Find the distance between each pair of points. Then find the coordinates of the midpoint of the line segment between the points.** (Lesson 1-3)

**49.** $A(2, 3)$, $B(5, 7)$

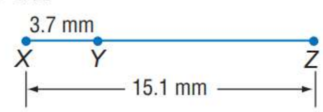
**50.** $C(-2, 0)$, $D(6, 4)$

**51.** $E(-3, -2)$, $F(5, 8)$

**Find the measurement of each segment.** (Lesson 1-2)

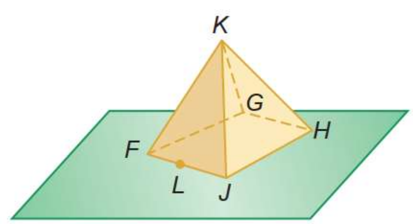
**52.** $\overline{WX}$


**53.** $\overline{YZ}$




**54.** Find $PQ$ if $Q$ lies between $P$ and $R$, $PQ = 6x - 5$, $QR = 2x + 7$, and $PQ = QR$. (Lesson 1-2)

**Refer to the figure at the right.** (Lesson 1-1)




**55.** How many planes are shown?

**56.** Name three collinear points.

**57.** Name a point coplanar with $J$, $H$, and $F$.

## GET READY for the Next Lesson

**PREREQUISITE SKILL** **Solve each equation.** (Pages 781–782)

**58.** $14x + (6x - 10) = 90$

**59.** $2k + 30 = 180$

**60.** $180 - 5y = 90 - 7y$

**61.** $90 - 4t = \frac{1}{4}(180 - t)$

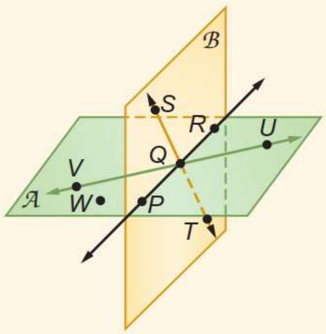
**62.** $(6m + 8) + (3m + 10) = 90$

**63.** $(7n - 9) + (5n + 45) = 180$

# Chapter 1 Mid-Chapter Quiz

Lessons 1-1 through 1-4

**For Exercises 1–2, refer to the figure.** (Lesson 1-1)



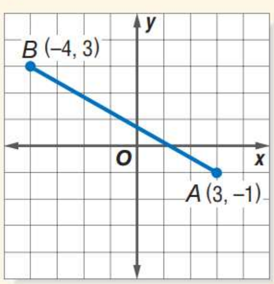

1. Name another point that is collinear with points $S$ and $Q$.
2. Name a line that is coplanar with $\overleftrightarrow{VU}$ and point $W$.

**Find the value of $x$ and $SR$ if $R$ is between $S$ and $T$.** (Lesson 1-2)

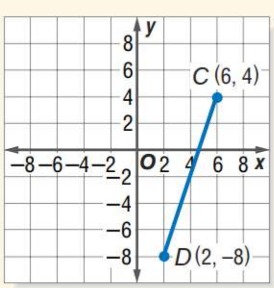
3. $SR = 3x$, $RT = 2x + 1$, $ST = 6x - 1$
4. $SR = 5x - 3$, $ST = 7x + 1$, $RT = 3x - 1$

**Find the coordinates of the midpoint of each segment. Then find the distance between the endpoints.** (Lesson 1-3)

5.




6.




**Find the coordinates of the midpoint of a segment having the given endpoints. Then find the distance between the endpoints.** (Lesson 1-3)

7. $E(10, 20)$, $F(-10, -20)$
8. $A(-1, 3)$, $B(5, -5)$
9. $C(4, 1)$, $D(-3, 7)$
10. $F(4, -9)$, $G(-2, -15)$
11. $H(-5, -2)$, $J(7, 4)$

12. **MULTIPLE CHOICE** $\overline{AB}$ has endpoints $A(n, 4n)$ and $B(3n, 6n)$. Which of the following is true?

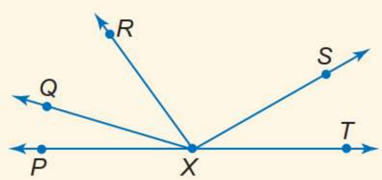
 **A** $AB = 4n$

 **B** The midpoint of $\overline{AB}$ is $(2n, 2n)$.

 **C** $AB = n\sqrt{8}$

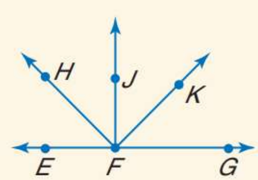
 **D** The midpoint of $\overline{AB}$ is $(4n, 10n)$.

**In the figure, $\overrightarrow{XP}$ and $\overrightarrow{XT}$ are opposite rays.** (Lesson 1-4)




13. If $m\angle SXT = 3a - 4$, $m\angle RXS = 2a + 5$, and $m\angle RXT = 111$, find $m\angle RXS$.
14. If $m\angle QXR = a + 10$, $m\angle QXS = 4a - 1$, and $m\angle RXS = 91$, find $m\angle QXS$.

**Measure each angle and classify as *right*, *acute*, or *obtuse*.** (Lesson 1-4)




15. $\angle KFG$
16. $\angle HFG$
17. $\angle HFK$
18. $\angle JFE$
19. $\angle HFJ$
20. $\angle EFK$

# 1-5 Angle Relationships

## Main Ideas

- Identify and use special pairs of angles.
- Identify perpendicular lines.

**Preparation for Standard 13.0** Students prove relationships between angles in polygons by using properties of complementary, supplementary, vertical, and exterior angles.

## New Vocabulary

adjacent angles
vertical angles
linear pair
complementary angles
supplementary angles
perpendicular

## GET READY for the Lesson

When two lines intersect, four angles are formed. In some cities, more than two streets might intersect to form even more angles. All of these angles are related in special ways.

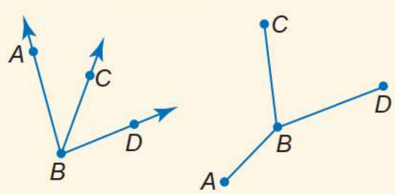
## Pairs of Angles

Certain pairs of angles have special names.

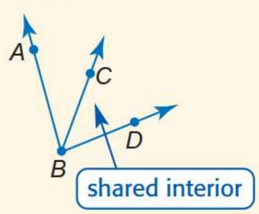
### KEY CONCEPT — Angle Pairs

**Words** **Adjacent angles** are two angles that lie in the same plane, have a common vertex and a common side, but no common interior points.

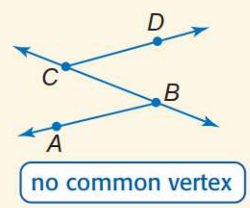
**Examples**
∠ABC and ∠CBD




**Nonexamples**
∠ABC and ∠ABD

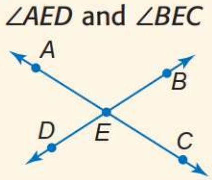



∠ABC and ∠BCD

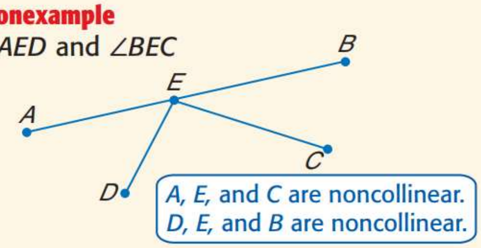



**Words** **Vertical angles** are two nonadjacent angles formed by two intersecting lines.

**Examples**
∠AEB and ∠CED
∠AED and ∠BEC

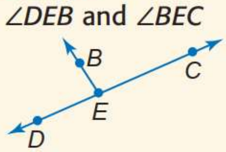



**Nonexample**
∠AED and ∠BEC

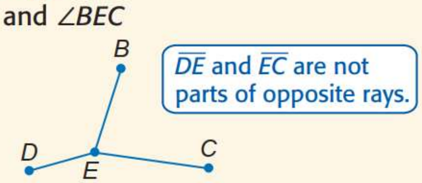



**Words** A **linear pair** is a pair of adjacent angles with noncommon sides that are opposite rays.

**Example**
∠DEB and ∠BEC

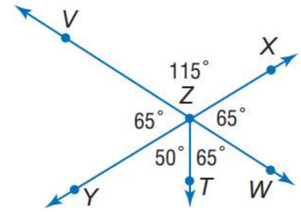



**Nonexample**
∠DEB and ∠BEC



## EXAMPLE 1 Identify Angle Pairs

**Name an angle pair that satisfies each condition.**




**a. two obtuse vertical angles**

$\angle VZX$ and $\angle YZW$ are vertical angles.

They each have measures greater than 90, so they are obtuse.

**b. two acute adjacent angles**

There are four acute angles shown.

Adjacent acute angles are $\angle VZY$ and $\angle YZT$, $\angle YZT$ and $\angle TZW$, and $\angle TZW$ and $\angle WZX$.

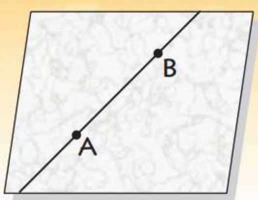
**CHECK Your Progress**

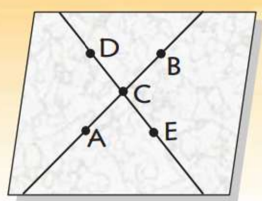
**1.** Name an angle pair that is a linear pair.

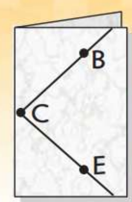
The measures of angles formed by intersecting lines have a special relationship.

## GEOMETRY LAB

### Angle Relationships




**Step 1**
Fold a piece of patty paper so that it makes a crease across the paper. Open the paper, trace the crease with a pencil, and name two points on the crease *A* and *B*.

**Step 2**
Fold the paper again so that the new crease intersects $\overleftrightarrow{AB}$ between the two labeled points. Open the paper, trace this crease, and label the intersection *C*. Label two other points, *D* and *E*, on the second crease so that *C* is between *D* and *E*.

**Step 3**
Fold the paper again through point *C* so that $\overrightarrow{CB}$ aligns with $\overrightarrow{CD}$.

#### ANALYZE THE MODEL

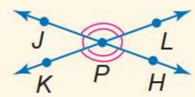
1. What did you notice about $\angle BCE$ and $\angle DCA$ when you made the last fold?
2. Fold again through *C* so that $\overrightarrow{CB}$ aligns with $\overrightarrow{CE}$. What do you notice?
3. Use a protractor to measure each angle. Label the measures on your model.
4. Name pairs of vertical angles and their measures.
5. Name linear pairs of angles and their measures.
6. Compare your results with those of your classmates. Write a "rule" about the measures of vertical angles and another about the measures of linear pairs.

The Geometry Lab suggests that all vertical angles are congruent. It also supports the concept that the sum of the measures of a linear pair is 180.

**KEY CONCEPT** — *Vertical Angles*

**Words** Vertical angles are congruent.

**Examples** $\angle JPK \cong \angle HPL$
$\angle JPL \cong \angle HPK$

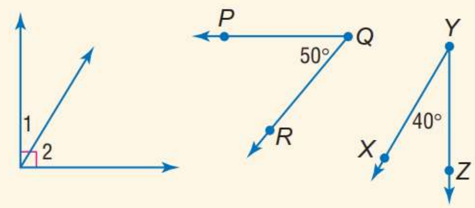



There are other angle relationships that you may remember from previous math courses. These are complementary angles and supplementary angles.

**KEY CONCEPT** — *Angle Relationships*

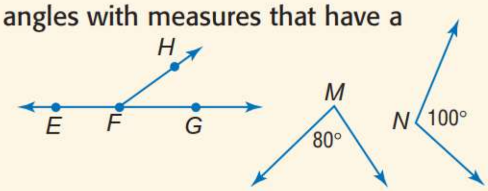
**Words** **Complementary angles** are two angles with measures that have a sum of 90.

**Examples**
$\angle 1$ and $\angle 2$ are complementary.
$\angle PQR$ and $\angle XYZ$ are complementary.




**Words** **Supplementary angles** are two angles with measures that have a sum of 180.

**Examples**
$\angle EFH$ and $\angle HFG$ are supplementary.
$\angle M$ and $\angle N$ are supplementary.




**Study Tip**

**Complementary and Supplementary Angles**

While the other angle pairs in this lesson share at least one point, complementary and supplementary angles need not share any points.

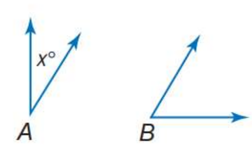
Remember that angle measures are real numbers. So, the operations for real numbers and algebra can be used with angle measures.

## EXAMPLE Angle Measure

**2** **ALGEBRA Find the measures of two complementary angles if the difference in the measures of the two angles is 12.**

**Explore** The problem relates the measures of two complementary angles. You know that the sum of the measures of complementary angles is 90.

**Plan** Draw two figures to represent the angles.
Let the measure of one angle be $x$.
If $m\angle A = x$, then, because $\angle A$ and $\angle B$ are complementary, $m\angle B + x = 90$ or $m\angle B = 90 - x$.




The problem states that the difference of the two angle measures is 12, or $m\angle B - m\angle A = 12$.

Online **Problem Solving Handbook at** ca.geometryonline.com

**Solve**

$$m\angle B - m\angle A = 12 \quad \text{Given}$$
$$(90 - x) - x = 12 \quad m\angle A = x,\ m\angle B = 90 - x$$
$$90 - 2x = 12 \quad \text{Simplify.}$$
$$-2x = -78 \quad \text{Subtract 90 from each side.}$$
$$x = 39 \quad \text{Divide each side by } -2.$$

Use the value of $x$ to find each angle measure.

$m\angle A = x$ | $m\angle B = 90 - x$

$m\angle A = 39$ | $m\angle B = 90 - 39$ or $51$

**Check** Add the angle measures to verify that the angles are complementary.

$$m\angle A + m\angle B = 90$$
$$39 + 51 = 90$$
$$90 = 90$$

**CHECK Your Progress**

**2.** Find the measures of two supplementary angles if the difference in the measures of the two angles is 32.

**Online Personal Tutor at** ca.geometryonline.com

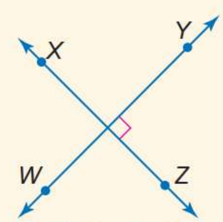
**Perpendicular Lines** Lines, segments, or rays that form right angles are **perpendicular**.

## KEY CONCEPT — *Perpendicular Lines*

**Words**

- Perpendicular lines intersect to form four right angles.
- Perpendicular lines intersect to form congruent adjacent angles.
- Segments and rays can be perpendicular to lines or to other line segments and rays.
- The right angle symbol in the figure indicates that the lines are perpendicular.

**Example**




$\overleftrightarrow{XZ} \perp \overleftrightarrow{WY}$

**Symbol** $\perp$ is read *is perpendicular to.*

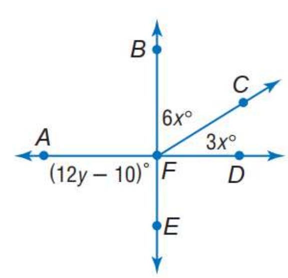
**Study Tip**

**Interpreting Figures**

Never assume that two lines are perpendicular because they appear to be so in the figure. The only sure way to know if they are perpendicular is if the right angle symbol is present or if the problem states angle measures that allow you to make that conclusion.

## EXAMPLE Perpendicular Lines

**3** **ALGEBRA Find $x$ and $y$ so that $\overrightarrow{BE}$ and $\overrightarrow{AD}$ are perpendicular.**




If $\overrightarrow{BE} \perp \overrightarrow{AD}$, then $m\angle BFD = 90$ and $m\angle AFE = 90$. To find $x$, use $\angle BFC$ and $\angle CFD$.

$$m\angle BFD = m\angle BFC + m\angle CFD \quad \text{Sum of parts = whole}$$
$$90 = 6x + 3x \quad \text{Substitution}$$
$$90 = 9x \quad \text{Add.}$$
$$10 = x \quad \text{Divide each side by 9.}$$

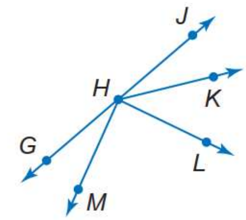
To find $y$, use $\angle AFE$.

| | |
|---|---|
| $m\angle AFE = 12y - 10$ | Given |
| $90 = 12y - 10$ | Substitution |
| $100 = 12y$ | Add 10 to each side. |
| $\frac{25}{3} = y$ | Divide each side by 12, and simplify. |

**CHECK Your Progress**

**3.** Suppose $m\angle D = 3x - 12$. Find $x$ so that $\angle D$ is a right angle.

While two lines may appear to be perpendicular in a figure, you cannot assume this is true unless other information is given. In geometry, figures are used to depict a situation. They are not drawn to reflect total accuracy of the situation. There are certain relationships you can assume to be true, but others that you cannot. Study the figure at the right and then compare the lists below.



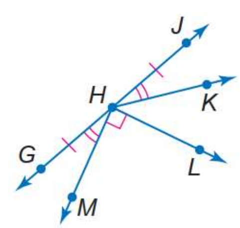

**Study Tip**

**Naming Figures**
The list of statements that can be assumed is not a complete list. There are more special pairs of angles than those listed. Also remember that all figures except points usually have more than one way to name them.

| Can Be Assumed | Cannot Be Assumed |
|---|---|
| All points shown are coplanar. | Perpendicular segments: $\overline{HL} \perp \overline{GJ}$ |
| $G$, $H$, and $J$ are collinear. | Congruent angles: $\angle JHK \cong \angle GHM$ |
| $\overrightarrow{HM}$, $\overrightarrow{HL}$, $\overrightarrow{HK}$, and $\overleftrightarrow{GJ}$ intersect at $H$. | $\angle JHK \cong \angle KHL$ |
| $H$ is between $G$ and $J$. | $\angle KHL \cong \angle GHM$ |
| $L$ is in the interior of $\angle MHK$. | Congruent segments: $\overline{GH} \cong \overline{HJ}$ |
| $\angle GHM$ and $\angle MHL$ are adjacent angles. | $\overline{HJ} \cong \overline{HK}$ |
| $\angle GHL$ and $\angle LHJ$ are a linear pair. | $\overline{HK} \cong \overline{HL}$ |
| $\angle JHK$ and $\angle KHG$ are supplementary. | $\overline{HL} \cong \overline{HG}$ |

## EXAMPLE 4 Interpret Figures

**Determine whether each statement can be assumed from the figure at the right.**




**a. $\angle GHM$ and $\angle MHK$ are adjacent angles.**

Yes; they share a common side and vertex and have no interior points in common.

**b. $\angle KHJ$ and $\angle GHM$ are complementary.**

No; they are congruent, but we do not know anything about their exact measures.

**c. $\angle GHK$ and $\angle JHK$ are a linear pair.**

Yes; they are adjacent angles whose noncommon sides are opposite rays.

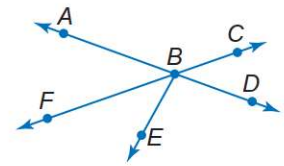
**CHECK Your Progress**

**4.** Determine whether the statement *$\angle GHL$ and $\angle LHJ$ are supplementary* can be assumed from the figure.

## CHECK Your Understanding

**Example 1** (p. 41)

**For Exercises 1 and 2, use the figure at the right and a protractor.**




**1.** Name two acute vertical angles.

**2.** Name two obtuse adjacent angles.

**Example 2** (pp. 42–43)

**3.** **SKIING** Alisa Camplin won a gold medal in the 2002 Winter Olympics with a triple-twisting, double backflip jump in the women's freestyle skiing event. While she is in the air, her skis give the appearance of intersecting lines. If $\angle 4$ measures 60°, find the measures of the other angles.

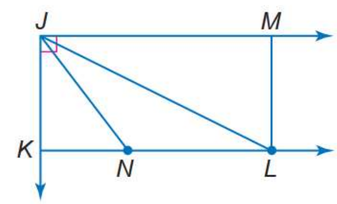

**Example 3** (pp. 43–44)

**4.** The measures of two complementary angles are $16z - 9$ and $4z + 3$. Find the measures of the angles.

**5.** Find $m\angle T$ if $m\angle T$ is 20 more than four times the measure of its supplement.

**Example 4** (p. 44)

**Determine whether each statement can be assumed from the figure. Explain.**




**6.** $\angle MLJ$ and $\angle JLN$ are complementary.

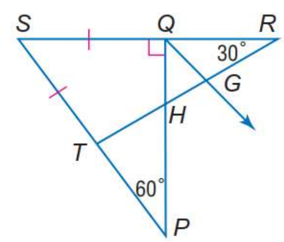
**7.** $\angle KJN$ and $\angle NJL$ are adjacent, but neither complementary nor supplementary.

## Exercises

**HOMEWORK HELP**

| For Exercises | See Examples |
|---|---|
| 8–13 | 1 |
| 14–19 | 2 |
| 20–22 | 3 |
| 23–27 | 4 |

**For Exercises 8–13, use the figure at the right and a protractor.**




**8.** Name two acute vertical angles.

**9.** Name two obtuse vertical angles.

**10.** Name a pair of complementary adjacent angles.

**11.** Name a pair of complementary nonadjacent angles.

**12.** Name a linear pair whose vertex is $G$.

**13.** Name an angle supplementary to $\angle HTS$.

**14.** Rays $PQ$ and $QR$ are perpendicular. Point $S$ lies in the interior of $\angle PQR$. If $m\angle PQS = 4 + 7a$ and $m\angle SQR = 9 + 4a$, find $m\angle PQS$ and $m\angle SQR$.

**15.** The measure of the supplement of an angle is 60 less than three times the measure of the complement of the angle. Find the measure of the angle.

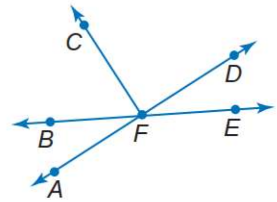
**16.** Lines $p$ and $q$ intersect to form adjacent angles 1 and 2. If $m\angle 1 = 3x + 18$ and $m\angle 2 = -8y - 70$, find the values of $x$ and $y$ so that $p$ is perpendicular to $q$.

**17.** The measure of an angle's supplement is 44 less than the measure of the angle. Find the measure of the angle and its supplement.

**18.** Two angles are supplementary. One angle measures 12° more than the other. Find the measures of the angles.

**19.** The measure of $\angle 1$ is five less than four times the measure of $\angle 2$. If $\angle 1$ and $\angle 2$ form a linear pair, what are their measures?

**ALGEBRA** **For Exercises 20–22, use the figure at the right.**

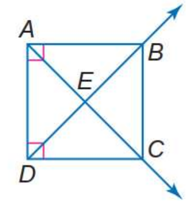



**20.** If $m\angle CFD = 12a + 45$, find $a$ so that $\overrightarrow{FC} \perp \overrightarrow{FD}$.

**21.** If $m\angle AFB = 8x - 6$ and $m\angle BFC = 14x + 8$, find the value of $x$ so that $\angle AFC$ is a right angle.

**22.** If $m\angle BFA = 3r + 12$ and $m\angle DFE = -8r + 210$, find $m\angle AFE$.

**Determine whether each statement can be assumed from the figure. Explain.**




**23.** $\angle DAB$ is a right angle.

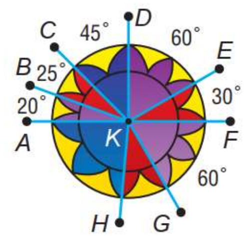
**24.** $\overline{AB} \perp \overline{BC}$

**25.** $\angle AEB \cong \angle DEC$

**26.** $\angle DAE \cong \angle ADE$

**27.** $\angle ADB$ and $\angle BDC$ are complementary.

**28. STAINED GLASS** In the stained glass pattern at the right, determine which segments are perpendicular.




**Determine whether each statement is *sometimes*, *always*, or *never* true.**

**29.** If two angles are supplementary and one is acute, the other is obtuse.

**30.** If two angles are complementary, they are both acute angles.

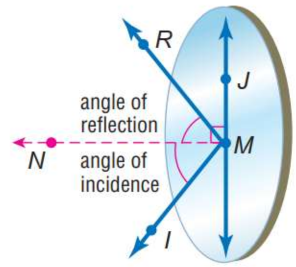
**31.** If $\angle A$ is supplementary to $\angle B$, and $\angle B$ is supplementary to $\angle C$, then $\angle A$ is supplementary to $\angle C$.

**32.** If $\overline{PN} \perp \overline{PQ}$, then $\angle NPQ$ is acute.

**Real-World Link**

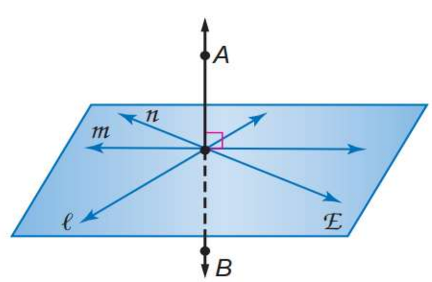
As light from the Sun travels to Earth, it is reflected or refracted by many different surfaces. Light that strikes a smooth, flat surface is very bright because the light is being reflected at the same angle.

**33. PHYSICS** As a ray of light meets a mirror, the light is reflected. The angle that the light strikes the mirror is the *angle of incidence*. The angle that the light is reflected is the *angle of reflection*. The angle of incidence and the angle of reflection are congruent. In the diagram at the right, if $m\angle RMI = 106$, find the angle of reflection and $m\angle RMJ$.




**34. RESEARCH** Look up the words *complementary* and *complimentary* in a dictionary. Discuss the differences in the terms and determine which word has a mathematical meaning.

**35.** The concept of perpendicularity can be extended to include planes. If a line, line segment, or ray is perpendicular to a plane, it is perpendicular to every line, line segment, or ray in that plane at the point of intersection. In the figure at the right, $\overleftrightarrow{AB} \perp \mathcal{E}$. Name all pairs of perpendicular lines.




**EXTRA PRACTICE**
See pages 801, 828.

**Math Online**
Self-Check Quiz at
ca.geometryonline.com

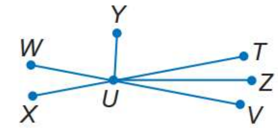
**H.O.T. Problems**

**36. OPEN ENDED** Draw two angles that are supplementary, but not adjacent.

**37. REASONING** Explain the statement *If two adjacent angles form a linear pair, they must be supplementary.*

**38. CHALLENGE** A *counterexample* is used to show that a statement is not necessarily true. Draw a counterexample for the statement *Supplementary angles form linear pairs.*

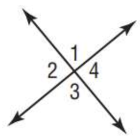
**39. CHALLENGE** In the figure, $\angle WUT$ and $\angle XUV$ are vertical angles, $\overline{YU}$ is the bisector of $\angle WUT$, and $\overline{UZ}$ is the bisector of $\angle TUV$. Write a convincing argument that $\overline{YU} \perp \overline{UZ}$.




**40. *Writing in Math*** Refer to page 40. What kinds of angles are formed when streets intersect? Include the types of angles that might be formed by two intersecting lines, and a sketch of intersecting streets with angle measures and angle pairs identified.

## STANDARDS PRACTICE

**41.** In the diagram below, $\angle 1$ is an acute angle.




Which conclusion is *not* true?

**A** $m\angle 2 > m\angle 3$

**B** $m\angle 2 = m\angle 4$

**C** $m\angle 1 < m\angle 4$

**D** $m\angle 3 > m\angle 4$

**42. REVIEW** Solve: $5(x - 4) = 3x + 18$

Step 1: $5x - 4 = 3x + 18$

Step 2: $2x - 4 = 18$

Step 3: $2x = 22$

Step 4: $x = 11$

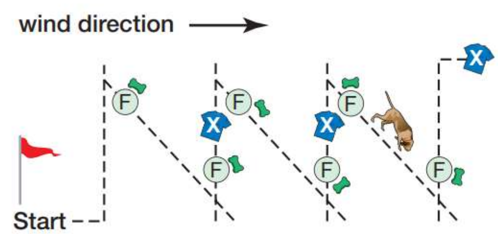
Which is the first *incorrect* step in the solution shown above?

**F** Step 1 **H** Step 3

**G** Step 2 **J** Step 4

## Spiral Review

**43. DOG TRACKING** A dog is tracking when it is following the scent trail left by a human being or other animal that has passed along a certain route. One of the training exercises for these dogs is a tracking trail. The one shown is called an acute tracking trail. Explain why it might be called this. (Lesson 1-4)




**Find the distance between each pair of points.** (Lesson 1-3)

**44.** $A(3, 5), B(0, 1)$ **45.** $C(5, 1), D(5, 9)$ **46.** $E(-2, -10), F(-4, 10)$

**47.** $G(7, 2), H(-6, 0)$ **48.** $J(-8, 9), K(4, 7)$ **49.** $L(1, 3), M(3, -1)$

**Find the value of the variable and *QR*, if *Q* is between *P* and *R*.** (Lesson 1-2)

**50.** $PQ = 1 - x, QR = 4x + 17, PR = -3x$ **51.** $PR = 7n + 8, PQ = 4n - 3, QR = 6n + 2$

## GET READY for the Next Lesson

**PREREQUISITE SKILL Evaluate each expression if $\ell = 3$, $w = 8$, and $s = 2$.** (Page 780)

**52.** $2\ell + 2w$ **53.** $\ell w$ **54.** $4s$ **55.** $\ell w + ws$ **56.** $s(\ell + w)$

EXTEND 1-5

Geometry Lab

# Constructing Perpendiculars

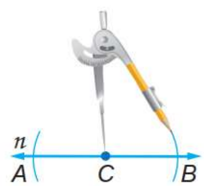
**Standard 16.0 Students perform basic constructions with a straightedge and compass,** such as angle bisectors, perpendicular bisectors, and the line parallel to a given line through a point off the line. (Key)

You can use a compass and a straightedge to construct a line perpendicular to a given line through a point on the line, or through a point not on the line.

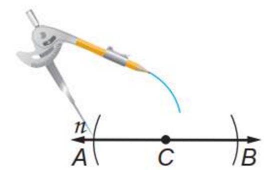
## ACTIVITY 1 Perpendicular Through a Point on the Line

**Construct a line perpendicular to line *n* and passing through point *C* on *n*.**

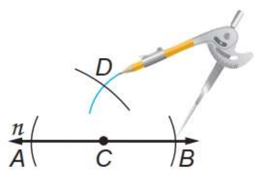
1. Place the compass at point *C*. Using the same compass setting, draw arcs to the right and left of *C*, intersecting line *n*. Label the points of intersection *A* and *B*.



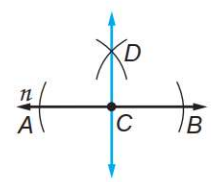

2. Open the compass to a setting greater than *AC*. Put the compass at point *A* and draw an arc above line *n*.




3. Using the same compass setting as in Step 2, place the compass at point *B* and draw an arc intersecting the arc drawn in Step 2. Label the point of intersection *D*.




4. Use a straightedge to draw $\overleftrightarrow{CD}$.

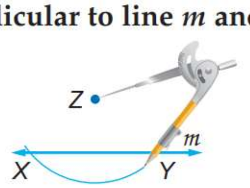



## ACTIVITY 2 Perpendicular Through a Point not on the Line

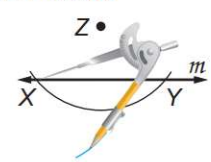
Concepts in Motion
Animation ca.geometryonline.com

**Construct a line perpendicular to line *m* and passing through point *Z* *not* on *m*.**

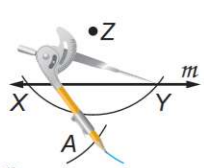
1. Place the compass at point *Z*. Draw an arc that intersects line *m* in two different places. Label the points of intersection *X* and *Y*.



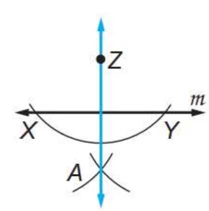

2. Open the compass to a setting greater than $\frac{1}{2}XY$. Put the compass at point *X* and draw an arc below line *m*.




3. Using the same compass setting, place the compass at point *Y* and draw an arc intersecting the arc drawn in Step 2. Label the point of intersection *A*.




4. Use a straightedge to draw $\overleftrightarrow{ZA}$.




### MODEL AND ANALYZE THE RESULTS

1. Draw a line and construct a line perpendicular to it through a point on the line. Repeat with a point not on the line.
2. How is the second construction similar to the first one?