Parallel Lines and Transversals

Main Ideas

- Identify the relationships between two lines or two planes.
- Name angles formed by a pair of lines and a transversal.

Preparation for Standard 7.0 Students prove and use theorems involving the properties of parallel lines cut by a transversal, the properties of quadrilaterals, and the properties of circles. (Key)

New Vocabulary

parallel lines
parallel planes
skew lines
transversal
consecutive interior angles
alternate exterior angles
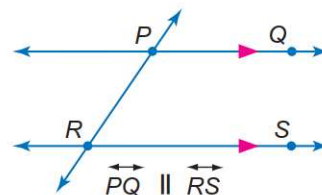alternate interior angles
corresponding angles

▶ GET READY for the Lesson

The south facade of the California Aerospace Museum in Los Angeles, California, was designed by Frank Gehry. There are several examples of parallel lines, parallel planes, and skew lines in the design.

Relationships Between Lines and Planes Lines ℓ and m are coplanar because they lie in the same plane. If the lines were extended indefinitely, they would not intersect. Coplanar lines that do not intersect are called **parallel lines**. Segments and rays contained within parallel lines are also parallel.

The symbol ∥ means *is parallel to*. Arrows are used in diagrams to indicate that lines are parallel. In the figure, the arrows indicate that $\overleftrightarrow{PQ}$ is parallel to $\overleftrightarrow{RS}$.

Similarly, two planes can intersect or be parallel. In the photograph above, the front faces of the building are contained in **parallel planes**. The walls and the floor of each level lie in intersecting planes.

$$\overleftrightarrow{PQ} \parallel \overleftrightarrow{RS}$$

The symbol ∦ means *is not parallel to.*
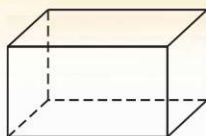
GEOMETRY LAB

Draw a Rectangular Prism

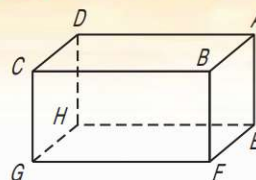**A rectangular prism can be drawn using parallel lines and parallel planes.**

Step 1 Draw two parallel planes to represent the top and bottom.

Step 2 Draw the edges. Make any hidden edges dashed.

Step 3 Label the vertices.

ANALYZE

1. Identify the parallel planes in the figure.
2. Name the planes that intersect plane *ABC* and name their intersections.
3. Identify all segments parallel to $\overline{BF}$.

Notice that in the Geometry Lab, $\overline{AE}$ and $\overline{GF}$ do not intersect. These segments are not parallel since they do not lie in the same plane. Lines that do not intersect and are not coplanar are called **skew lines**. Segments and rays contained in skew lines are also skew.

EXAMPLE   Identify Relationships

1

a. Name all planes that are parallel to plane *ABG*.
plane *CDE*

b. Name all segments that intersect $\overline{CH}$.
$\overline{BC}, \overline{CD}, \overline{CE}, \overline{EH}$, and $\overline{GH}$

c. Name all segments that are skew to $\overline{BG}$.
$\overline{AD}, \overline{CD}, \overline{CE}, \overline{EF}$, and $\overline{EH}$
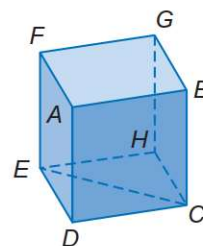
CHECK Your Progress
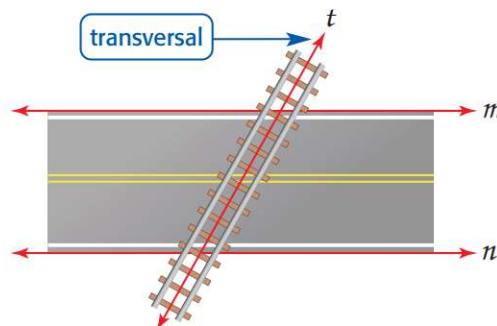**1.** Name all segments that are parallel to $\overline{EF}$.

Angle Relationships In the drawing of the railroad crossing, notice that the tracks, represented by line *t*, intersect the sides of the road, represented by lines *m* and *n*. A line that intersects two or more lines in a plane at different points is called a **transversal**.
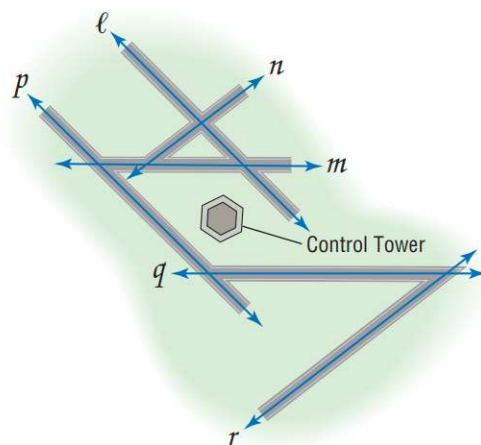
Real-World EXAMPLE   Identify Transversals

2

AIRPORTS Some of the runways at O'Hare International Airport are shown below. Identify the sets of lines to which each given line is a transversal.

a. line *q*
If the lines are extended, line *q* intersects lines *ℓ, n, p,* and *r*.

b. line *m*
lines *ℓ, n, p,* and *r*

c. line *n*
lines *ℓ, m, p,* and *q*

CHECK Your Progress
2. line *r*

In the drawing of the railroad crossing above, notice that line *t* forms eight angles with lines *m* and *n*. These angles are given special names, as are specific pairings of these angles.

| KEY **CONCEPT**		*Transversals and Angles*
Name	**Angles**	Transversal *p* intersects lines *q* and *r*.
exterior angles	∠1, ∠2, ∠7, ∠8	
interior angles	∠3, ∠4, ∠5, ∠6	
consecutive interior angles	∠3 and ∠6, ∠4 and ∠5	
alternate exterior angles	∠1 and ∠7, ∠2 and ∠8	
alternate interior angles	∠3 and ∠5, ∠4 and ∠6	
corresponding angles	∠1 and ∠5, ∠2 and ∠6, ∠3 and ∠7, ∠4 and ∠8	
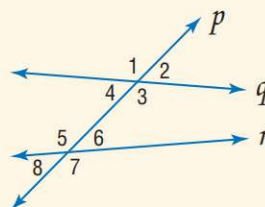
EXAMPLE **Identify Angle Relationships**

3 Refer to the figure below. Identify each pair of angles as *alternate interior, alternate exterior, corresponding,* or *consecutive interior* angles.

a. ∠1 and ∠7
alternate exterior

b. ∠2 and ∠10
corresponding

c. ∠8 and ∠9
consecutive interior

d. ∠3 and ∠12
corresponding

e. ∠4 and ∠10
alternate interior

f. ∠6 and ∠11
alternate exterior
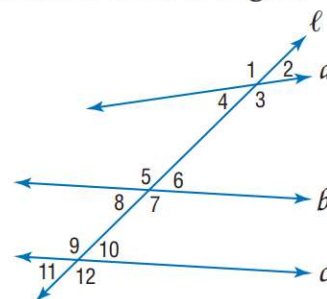
CHECK Your Progress

3A. ∠4 and ∠11
3B. ∠2 and ∠8

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CHECK Your Understanding

Example 1
(p. 143)

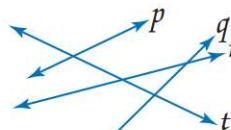For Exercises 1–3, refer to the figure at the right.
1. Name all planes that intersect plane *ADM*.
2. Name all segments that are parallel to $\overline{CD}$.
3. Name all segments that intersect $\overline{KL}$.
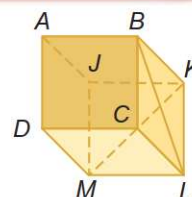
Example 2
(p. 143)

Identify the pairs of lines to which each given line is a transversal.
4. *p*   **5.** *r*
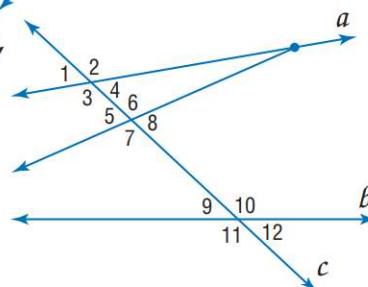**6.** *q*   **7.** *t*

Example 3
(p. 144)

Identify each pair of angles as *alternate interior, alternate exterior, corresponding,* or *consecutive interior* angles.
8. ∠7 and ∠10
9. ∠1 and ∠5
10. ∠4 and ∠6
11. ∠8 and ∠1

For Exercises 12–19, refer to the figure at the right.

12. Name all segments parallel to $\overline{AB}$.

13. Name all planes intersecting plane BCR.

14. Name all segments parallel to $\overline{TW}$.

15. Name all segments skew to $\overline{DE}$.

16. Name all planes intersecting plane EDS.

17. Name all segments skew to $\overline{AP}$.

18. Name all segments parallel to $\overline{DC}$.

19. Name all segments parallel to $\overline{DS}$.

Identify the pairs of lines to which each given line is a transversal.

20. a **21.** b

22. c **23.** r

Identify each pair of angles as *alternate interior, alternate exterior, corresponding,* or *consecutive interior angles.*

24. $\angle 2$ and $\angle 10$ **25.** $\angle 1$ and $\angle 11$

26. $\angle 5$ and $\angle 3$ **27.** $\angle 6$ and $\angle 14$

28. $\angle 5$ and $\angle 15$ **29.** $\angle 11$ and $\angle 13$

30. $\angle 8$ and $\angle 3$ **31.** $\angle 9$ and $\angle 4$

32. $\angle 6$ and $\angle 16$ **33.** $\angle 7$ and $\angle 3$

34. $\angle 10$ and $\angle 13$ **35.** $\angle 12$ and $\angle 14$

36. AVIATION Airplanes heading east are assigned an altitude level that is an odd number of thousands of feet. Airplanes heading west are assigned an altitude level that is an even number of thousands of feet. If one airplane is flying northwest at 34,000 feet and another airplane is flying east at 25,000 feet, describe the type of lines formed by the paths of the airplanes. Explain your reasoning.

Real-World Career

Air Traffic Controller

The air traffic controller monitors and coordinates air traffic either at an airport or between airports. They monitor the position of airplanes ensuring that planes are a safe distance apart.

Math Online

For more information, go to ca.geometryonline.com.

MONUMENTS For Exercises 37–40, refer to the photograph of the Lincoln Memorial.

37. Describe a pair of parallel lines found on the Lincoln Memorial.

38. Find an example of parallel planes.

39. Locate a pair of skew lines.

40. Identify a transversal passing through a pair of lines.

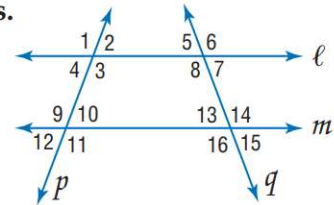**Name the transversal that forms each pair of angles. Then identify the special name for the angle pair.**

41. ∠3 and ∠10

**42.** ∠2 and ∠12

43. ∠8 and ∠14

44. ∠9 and ∠16

STRUCTURES For Exercises 45-47, refer to the drawing of the gazebo at the right.

45. Name all labeled segments parallel to $\overline{BF}$.

46. Name all labeled segments skew to $\overline{AC}$.

**47.** Are any of the planes on the gazebo parallel to plane *ADE*? Explain.
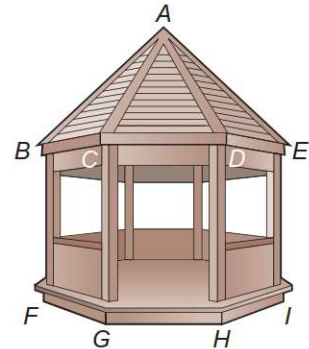
48. **RESEARCH** The word *parallel* describes computer processes that occur simultaneously, or devices, such as printers, that receive more than one bit of data at a time. Find two other examples for uses of the word *parallel* in other subject areas such as history, music, or sports.
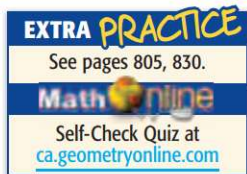
H.O.T. Problems

49. **OPEN ENDED** Draw a solid figure with parallel planes. Describe which parts of the figure are parallel.

50. **FIND THE ERROR** Juanita and Eric are naming alternate interior angles in the figure at the right. One of the angles must be ∠4. Who is correct? Explain your reasoning.
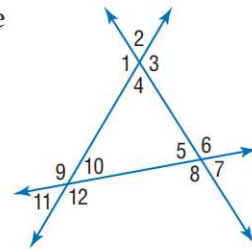
Juanita

∠4 and ∠9

∠4 and ∠6

Eric

∠4 and ∠10

∠4 and ∠5

CHALLENGE Suppose there is a line ℓ and a point *P* not on the line.

51. In space, how many lines can be drawn through *P* that do not intersect ℓ?

52. In space, how many lines can be drawn through *P* that are parallel to ℓ?

53. *Writing in Math* Use the information about architecture on page 142 to explain how parallel lines and planes are used in architecture. Include a description of where you might find examples of parallel lines and parallel planes, and skew lines and nonparallel planes.

54. Find the pair of alternate exterior angles in the figure.
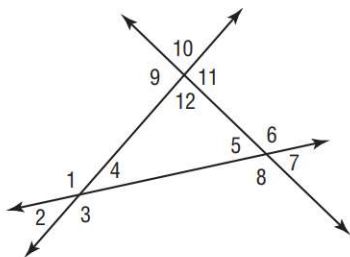
A ∠1 and ∠5

B ∠2 and ∠10

C ∠2 and ∠6

D ∠5 and ∠9

55. REVIEW Which coordinate points represent the *x*- and *y*-intercepts of the graph shown below?
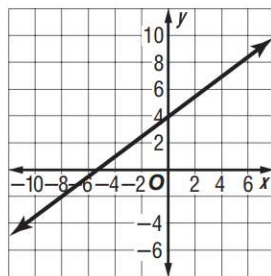
F $(-5.6, 0), (0, 4)$

G $(5.6, 0), (4, 0)$

H $(6, 0), (0, 4)$

J $(0, 4), (0, 6)$

Spiral Review

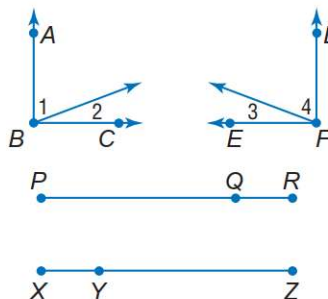**56. PROOF** Write a two-column proof. (Lesson 2-8)

    Given: $m\angle ABC = m\angle DFE, m\angle 1 = m\angle 4$

    Prove: $m\angle 2 = m\angle 3$

57. PROOF Write a paragraph proof. (Lesson 2-7)

    Given: $\overline{PQ} \cong \overline{ZY}, \overline{QR} \cong \overline{XY}$

    Prove: $\overline{PR} \cong \overline{XZ}$

Determine whether a valid conclusion can be reached from the two true statements using the Law of Detachment or the Law of Syllogism. If a valid conclusion is possible, state it and the law that is used. If a valid conclusion does not follow, write *no conclusion*. (Lesson 2-4)

58. (1) If two angles are vertical, then they do not form a linear pair.

    (2) If two angles form a linear pair, then they are not congruent.

59. (1) If an angle is acute, then its measure is less than 90.

    (2) ∠*EFG* is acute.

GET READY for the Next Lesson

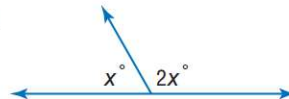**PREREQUISITE SKILL** State the measures of linear pairs of angles in each figure. (Lesson 2-6)

60.
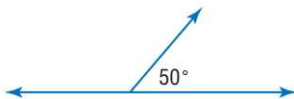
61.
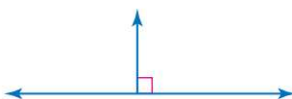
62.

Geometry Software Lab
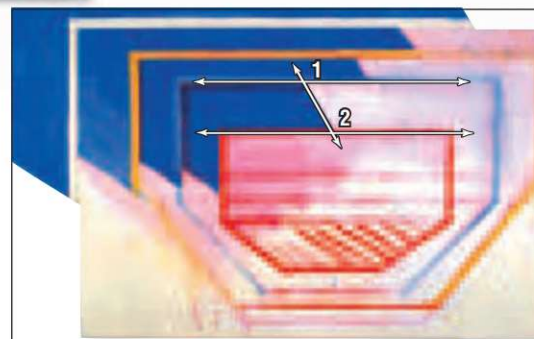# Angles and Parallel Lines

Preparation for Standard 7.0 Students prove and use theorems involving the properties of parallel lines cut by a transversal, the properties of quadrilaterals, and the properties of circles. (Key)

You can use The Geometer's Sketchpad® to investigate the measures of angles formed by two parallel lines and a transversal.

ACTIVITY

Step 1 Draw parallel lines.
- Construct points A and B.
- Construct a line through the points.
- Place point C so that it does not lie on $\overleftrightarrow{AB}$.
- Construct a line through C parallel to $\overleftrightarrow{AB}$.
- Place point D on this line.
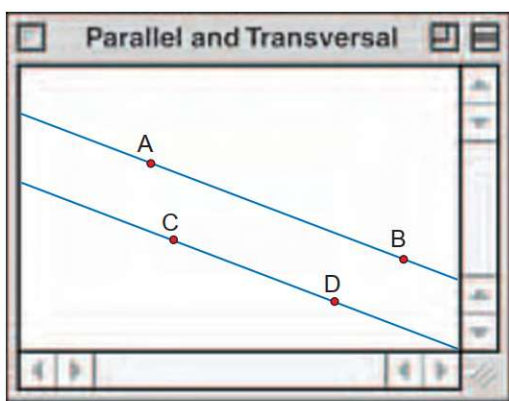
Step 2 Construct a transversal.
- Place point E on $\overleftrightarrow{AB}$ and point F on $\overleftrightarrow{CD}$.
- Construct a line through points E and F.
- Place points G and H on $\overleftrightarrow{EF}$.

Step 3 Measure angles.
- Measure each angle.
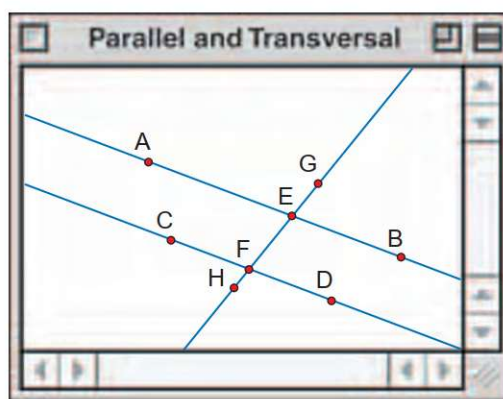
ANALYZE THE RESULTS

1. List pairs of angles by the special names you learned in Lesson 3-1. Which pairs have the same measure?

2. What is the relationship between consecutive interior angles?

3. **Make a conjecture** about the following pairs of angles formed by two parallel lines and a transversal. Write your conjecture in if-then form.
   a. corresponding angles
   b. alternate interior angles
   c. alternate exterior angles
   d. consecutive interior angles

4. Rotate the transversal. Are the angles with equal measures in the same relative location as the angles with equal measures in your original drawing?

5. Test your conjectures by rotating the transversal and analyzing the angles.

6. Rotate the transversal so that the measure of at least one angle is 90.
   a. What do you notice about the measures of the other angles?
   b. Make a conjecture about a transversal that is perpendicular to one of two parallel lines.

Main Ideas

- Use the properties of parallel lines to determine congruent angles.
- Use algebra to find angle measures.

Standard 7.0 Students prove and use theorems involving the properties of parallel lines cut by a transversal, the properties of quadrilaterals, and the properties of circles. (Key)

GET READY for the Lesson

In the painting, the artist uses lines and transversals to create patterns. The figure on the painting shows two parallel lines with a transversal passing through them. There is a special relationship between the angle pairs formed by these lines.

Source: *Vista* by Richard Smith

Parallel Lines and Angle Pairs In the figure above, ∠1 and ∠2 are corresponding angles. When the two lines are parallel, there is a special relationship between these pairs of angles.

POSTULATE 3.1 — Corresponding Angles Postulate

If two parallel lines are cut by a transversal, then each pair of corresponding angles is congruent.

Examples: ∠1 ≅ ∠5, ∠2 ≅ ∠6, ∠3 ≅ ∠7, ∠4 ≅ ∠8
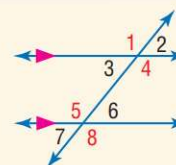
Review Vocabulary

Vertical Angles two nonadjacent angles formed by two intersecting lines (Lesson 1-6)

EXAMPLE — Determine Angle Measures

1 In the figure, $m\angle 3 = 133$. Find $m\angle 5$.

∠3 ≅ ∠7	Corresponding Angles Postulate
∠7 ≅ ∠5	Vertical Angles Theorem
∠3 ≅ ∠5	Transitive Property
$m\angle 3 = m\angle 5$	Definition of congruent angles
$133 = m\angle 5$	Substitution
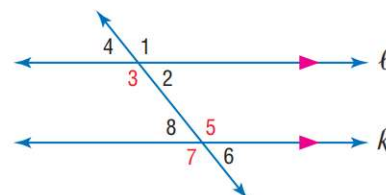
CHECK Your Progress

1. In the figure, $m\angle 8 = 47$. Find $m\angle 4$.

In Example 1, alternate interior angles 3 and 5 are congruent. This suggests another special relationship between angles formed by two parallel lines and a transversal. Other relationships are summarized in Theorems 3.1, 3.2, and 3.3.

THEOREM — Parallel Lines and Angle Pairs

Theorems	Examples	Model
3.1 Alternate Interior Angles If two parallel lines are cut by a transversal, then each pair of alternate interior angles is congruent.	$\angle 4 \cong \angle 5$ $\angle 3 \cong \angle 6$	
3.2 Consecutive Interior Angles If two parallel lines are cut by a transversal, then each pair of consecutive interior angles is supplementary.	$\angle 4$ and $\angle 6$ are supplementary. $\angle 3$ and $\angle 5$ are supplementary.	
3.3 Alternate Exterior Angles If two parallel lines are cut by a transversal, then each pair of alternate exterior angles is congruent.	$\angle 1 \cong \angle 8$ $\angle 2 \cong \angle 7$	

You will prove Theorems 3.2 and 3.3 in Exercises 26 and 23, respectively.

Proof · Theorem 3.1

Given: $a \parallel b$; p is a transversal of a and b.

Prove: $\angle 2 \cong \angle 7$, $\angle 3 \cong \angle 6$

Paragraph Proof: We are given that $a \parallel b$ with a transversal p. By the Corresponding Angles Postulate, $\angle 2 \cong \angle 4$ and $\angle 8 \cong \angle 6$. Also, $\angle 4 \cong \angle 7$ and $\angle 3 \cong \angle 8$ because vertical angles are congruent. Therefore, $\angle 2 \cong \angle 7$ and $\angle 3 \cong \angle 6$ since congruence of angles is transitive.
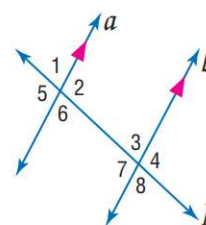
A special relationship occurs when the transversal is a perpendicular line.

THEOREM 3.4 — Perpendicular Transversal Theorem

In a plane, if a line is perpendicular to one of two parallel lines, then it is perpendicular to the other.
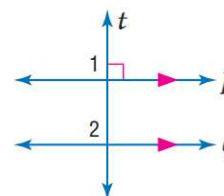
Proof · Theorem 3.4

Given: $p \parallel q, t \perp p$

Prove: $t \perp q$

Proof:

Statements	Reasons
1. $p \parallel q, t \perp p$	1. Given
2. $\angle 1$ is a right angle.	2. Definition of $\perp$ lines
3. $m\angle 1 = 90$	3. Definition of right angle
4. $\angle 1 \cong \angle 2$	4. Corresponding Angles Postulate
5. $m\angle 1 = m\angle 2$	5. Definition of congruent angles
6. $m\angle 2 = 90$	6. Substitution Property
7. $\angle 2$ is a right angle.	7. Definition of right angles
8. $t \perp q$	8. Definition of $\perp$ lines

2 What is $m\angle GHI$?

A 50°	**C** 130°
B 110°	**D** 140°
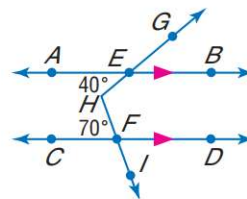
Read the Item

You need to find $m\angle GHI$.

Solve the Item

Draw $\overleftrightarrow{JK}$ through H parallel to $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$.

$\angle EHK \cong \angle AEH$ Alternate Interior Angles Theorem

$m\angle EHK = m\angle AEH$ Definition of congruent angles

$m\angle EHK = 40$ Substitution

$\angle FHK \cong \angle CFH$ Alternate Interior Angles Theorem

$m\angle FHK = m\angle CFH$ Definition of congruent angles

$m\angle FHK = 70$ Substitution

$m\angle GHI = m\angle EHK + m\angle FHK$ Angle Addition Postulate

$\qquad = 40 + 70$ or 110 $m\angle EHK = 40, m\angle FHK = 70$

Thus, the answer is choice B.
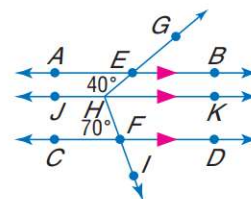
CHECK Your Progress

2. What is $m\angle 1$?

F 45°	**H** 90°
G 65°	**J** 135°
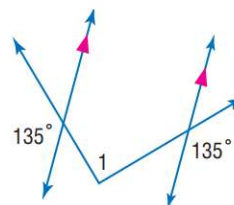
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Algebra and Angle Measures Angles formed by two parallel lines and a transversal can be used to find unknown values.

EXAMPLE   **Find Values of Variables**

3 **ALGEBRA** If $m\angle 1 = 3x + 40$ and $m\angle 3 = 2x + 70$, find x.

Since $\overleftrightarrow{FG} \parallel \overleftrightarrow{EH}$, $\angle 1 \cong \angle 3$ by the Corresponding Angles Postulate.

$m\angle 1 = m\angle 3$ Definition of congruent angles

$3x + 40 = 2x + 70$ Substitution

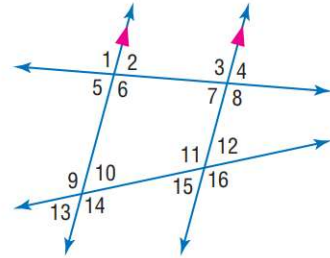$\qquad x = 30$ Subtract $2x$ and 40 from each side.
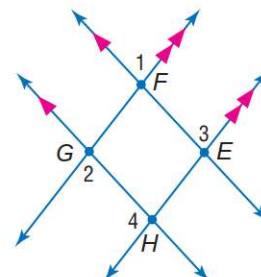
CHECK Your Progress

3. Refer to the figure. If $m\angle 2 = 4x + 7$ and $m\angle 3 = 5x - 13$, find $m\angle 3$.

Example 1
(p. 149)

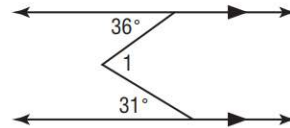In the figure, $m\angle 3 = 110$ and $m\angle 12 = 55$. Find the measure of each angle.

1. $\angle 1$ **2.** $\angle 6$ **3.** $\angle 2$
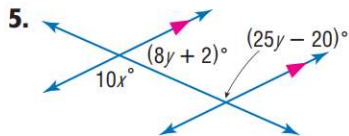
Example 2
(p. 151)

4. 🌀 **STANDARDS PRACTICE** What is $m\angle 1$?

A $5°$ **C** $36°$

B $31°$ **D** $67°$

Example 3
(p. 151)

Find x and y in each figure.

5. $(8y + 2)°$ $(25y - 20)°$ $10x°$

6. $(3y + 1)°$ $(4x - 5)°$ $(3x + 11)°$

Exercises

HOMEWORK HELP	
For Exercises	**See Examples**
7–12	1
13–18	2
19, 20	3

In the figure, $m\angle 3 = 43$. Find the measure of each angle.

7. $\angle 2$ **8.** $\angle 7$

9. $\angle 10$ **10.** $\angle 11$
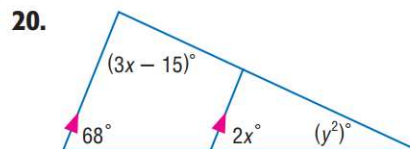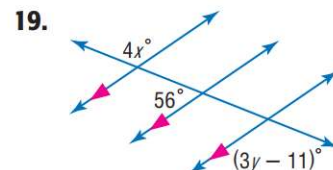
11. $\angle 13$ **12.** $\angle 16$
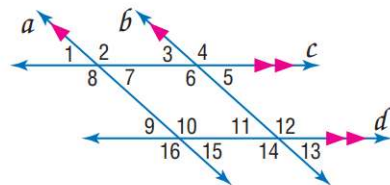
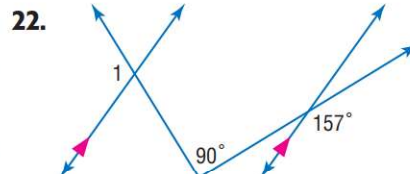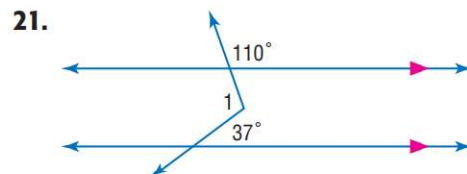In the figure, $m\angle 1 = 50$ and $m\angle 3 = 60$. Find the measure of each angle.

13. $\angle 4$ **14.** $\angle 5$

15. $\angle 2$ **16.** $\angle 6$

17. $\angle 7$ **18.** $\angle 8$
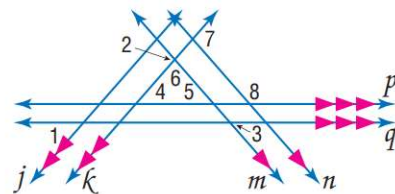
Find x and y in each figure.

19. $4x°$ $56°$ $(3y - 11)°$

20. $(3x - 15)°$ $68°$ $2x°$ $(y^2)°$

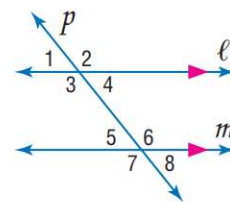Find $m\angle 1$ in each figure.

21. $110°$ 1 $37°$

22. 1 $90°$ $157°$
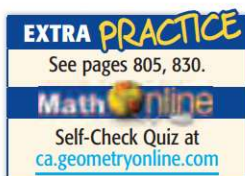
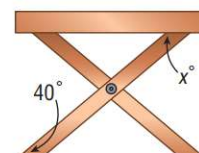**23. PROOF** Copy and complete the proof of Theorem 3.3.
Given: $\ell \parallel m$
Prove: $\angle 1 \cong \angle 8$
 $\angle 2 \cong \angle 7$
Proof:

Statements	Reasons
1. $\ell \parallel m$	1. _____?_____
2. $\angle 1 \cong \angle 5$, $\angle 2 \cong \angle 6$	2. _____?_____
3. $\angle 5 \cong \angle 8$, $\angle 6 \cong \angle 7$	3. _____?_____
4. $\angle 1 \cong \angle 8$, $\angle 2 \cong \angle 7$	4. _____?_____

24. CARPENTRY Anthony is building a picnic table for his patio. He cut one of the legs at an angle of 40°. At what angle should he cut the other end to ensure that the top of the table is parallel to the ground? Explain.

25. CONSTRUCTION Parallel drainage pipes are connected with a third pipe. The connector pipe makes a 65° angle with a pipe as shown. What is the measure of the angle it makes with the pipe on the other side of the road? Explain.

connector pipe

pipe

65°
?°
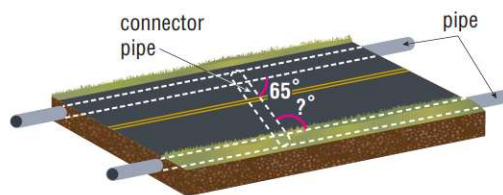
26. PROOF Write a two-column proof of Theorem 3.2.

Refer to the figure for Exercises 27 and 28.
27. Determine whether $\angle 1$ is *always*, *sometimes*, or *never* congruent to $\angle 2$. Explain.
28. Determine the minimum number of angle measures you would have to know to find the measures of all of the angles in the figure.

H.O.T. Problems
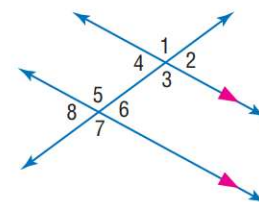
29. OPEN ENDED Use a straightedge and protractor to draw a pair of parallel lines cut by a transversal so that one pair of corresponding angles measures 35°.

30. REASONING Make a conjecture about two exterior angles on the same side of a transversal. Prove your conjecture.

31. CHALLENGE Explain why you can conclude that $\angle 2$ and $\angle 6$ are supplementary, but you cannot state that $\angle 4$ and $\angle 6$ are necessarily supplementary.
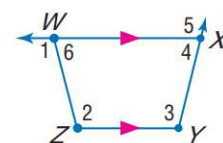
32. *Writing in Math* Use the information about art from page 149 to explain how angles and lines can be used in art. Include a description of how angles and lines are used to create patterns and examples from two different artists that use lines and angles.

33. In the accompanying diagram, parallel lines ℓ and m are cut by transversal p.
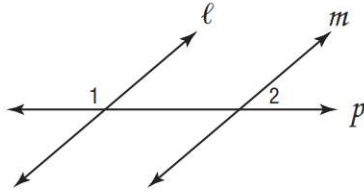
Which statement about angles 1 and 2 *must* be true?

A $\angle 1 \cong \angle 2$

B $\angle 1$ is the complement of $\angle 2$.

C $\angle 1$ and $\angle 2$ are acute angles.

D $\angle 1$ is the supplement of $\angle 2$.

34. REVIEW Emma has been hiring more workers for her donut shop. The table shows the number of additional workers compared to the number of donuts the shop can make in an hour.

| Additional Workers	Donuts Made
0	45
1	70
2	95
3	120

Which equation best describes the relationship between w, the number of additional workers, and d, the number of donuts the shop can make in an hour?

F $45w + 25 = d$

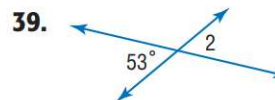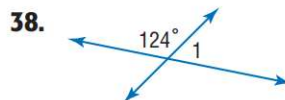**G** $d - 45 = 25w$

H $d + 45 = 25w$

J $45w - 25 = d$

Spiral Review

For Exercises 35–37, refer to the figure at the right. (Lesson 3-1)

35. Name all segments parallel to $\overline{AB}$.

36. Name all segments skew to $\overline{CH}$.

37. Name all planes parallel to AEF.
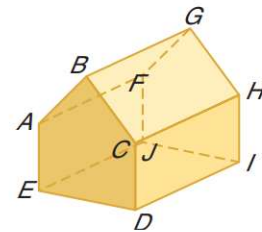
Find the measure of each numbered angle. (Lesson 2-8)

38.

39.

Identify the hypothesis and conclusion of each statement. (Lesson 2-3)

40. If it rains this evening, then I will mow the lawn tomorrow.

41. A balanced diet will keep you healthy.

PREREQUISITE SKILL Simplify each expression.

42. $\dfrac{7-9}{8-5}$ **43.** $\dfrac{-3-6}{2-8}$ **44.** $\dfrac{14-11}{23-15}$ **45.** $\dfrac{15-23}{14-11}$ **46.** $\dfrac{2}{9} \cdot \left(-\dfrac{18}{5}\right)$