4-1 Classifying Triangles

Main Ideas

- Identify and classify triangles by angles.
- Identify and classify triangles by sides.

Standard 12.0 **Students find and use measures of sides and of interior** and exterior **angles of triangles** and polygons to **classify figures and solve problems.** (Key)

New Vocabulary

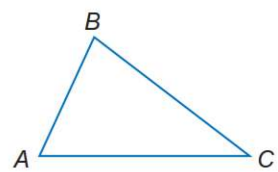
acute triangle
obtuse triangle
right triangle
equiangular triangle
scalene triangle
isosceles triangle
equilateral triangle

GET READY for the Lesson

Many structures use triangular shapes as braces for construction. The roof sections of houses are made of triangular trusses that support the roof and the house.

Classify Triangles by Angles Triangle *ABC*, written $\triangle ABC$, has parts that are named using the letters *A*, *B*, and *C*.

- The sides of $\triangle ABC$ are $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$.
- The vertices are *A*, *B*, and *C*.
- The angles are $\angle ABC$ or $\angle B$, $\angle BCA$ or $\angle C$, and $\angle BAC$ or $\angle A$.



There are two ways to classify triangles. One way is by their angles. All triangles have at least two acute angles, but the third angle is used to classify the triangle.

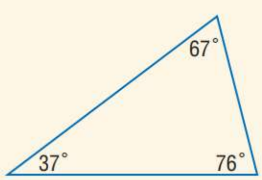
Study Tip

Common Misconceptions

It is a common mistake to classify triangles by their angles in more than one way. These classifications are distinct groups. For example, a triangle cannot be right and acute.

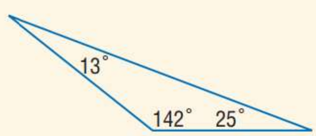
KEY CONCEPT *Classifying Triangles by Angle*

In an **acute triangle**, all of the angles are acute.



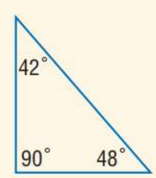
all angle measures < 90

In an **obtuse triangle**, one angle is obtuse.



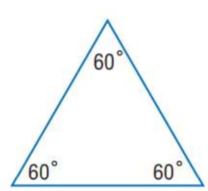
one angle measure > 90

In a **right triangle**, one angle is right.

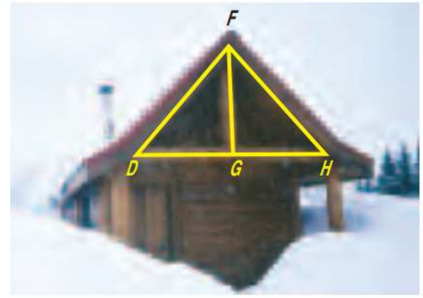


one angle measure = 90

An acute triangle with all angles congruent is an **equiangular triangle**.



Real-World EXAMPLE — Classify Triangles by Angles



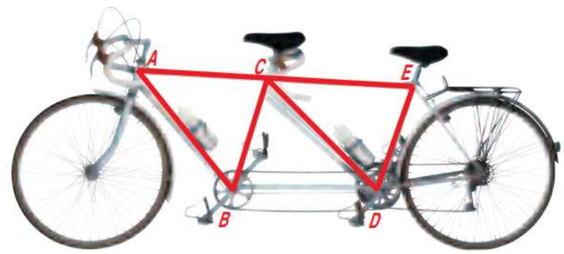
1 **ARCHITECTURE** **The roof of this house is made up of three different triangles. Use a protractor to classify △*DFH*, △*DFG*, and △*HFG* as *acute*, *equiangular*, *obtuse*, or *right*.**

△*DFH* has all angles with measures less than 90, so it is an acute triangle. △*DFG* and △*HFG* both have one angle with measure equal to 90. Both of these are right triangles.

Study Tip

Congruency
To indicate that sides of a triangle are congruent, an equal number of hash marks are drawn on the corresponding sides.

CHECK Your Progress



1. BICYCLES The frame of this tandem bicycle uses triangles. Use a protractor to classify △*ABC* and △*CDE*.

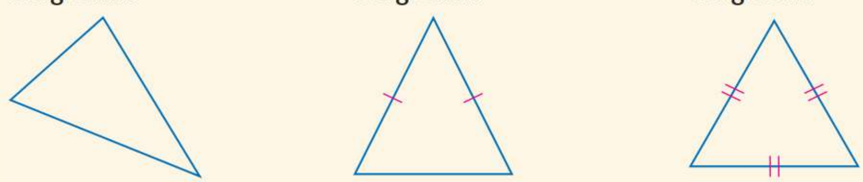
Classify Triangles by Sides Triangles can also be classified according to the number of congruent sides they have.

Study Tip

Equilateral Triangles
An equilateral triangle is a special kind of isosceles triangle.

KEY CONCEPT — *Classifying Triangles by Sides*

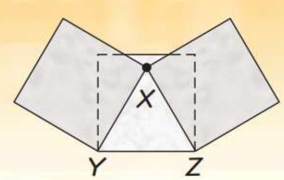
No two sides of a **scalene triangle** are congruent.	At least two sides of an **isosceles triangle** are congruent.	All of the sides of an **equilateral triangle** are congruent.



GEOMETRY LAB

Equilateral Triangles

MODEL



- Align three pieces of patty paper. Draw a dot at *X*.
- Fold the patty paper through *X* and *Y* and through *X* and *Z*.

ANALYZE

1. Is △*XYZ* equilateral? Explain.
2. Use three pieces of patty paper to make a triangle that is isosceles, but not equilateral.
3. Use three pieces of patty paper to make a scalene triangle.

EXAMPLE Classify Triangles by Sides

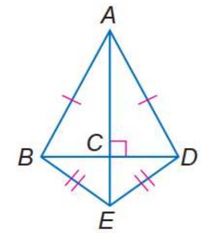
2 **Identify the indicated type of triangle in the figure.**

a. isosceles triangles

Isosceles triangles have at least two sides congruent. So, $\triangle ABD$ and $\triangle EBD$ are isosceles.

b. scalene triangles

Scalene triangles have no congruent sides. $\triangle AEB$, $\triangle AED$, $\triangle ACB$, $\triangle ACD$, $\triangle BCE$, and $\triangle DCE$ are scalene.

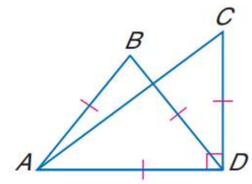


CHECK Your Progress

2 **Identify the indicated type of triangle in the figure.**

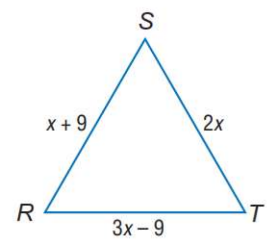
2A. equilateral

2B. isosceles



EXAMPLE Find Missing Values

3 **ALGEBRA** **Find x and the measure of each side of equilateral triangle RST.**



Since $\triangle RST$ is equilateral, $RS = ST$.

$x + 9 = 2x$ Substitution

$9 = x$ Subtract x from each side.

Next, substitute to find the length of each side.

$RS = x + 9$
$= 9 + 9$ or 18

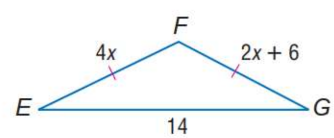
$ST = 2x$
$= 2(9)$ or 18

$RT = 3x - 9$
$= 3(9) - 9$ or 18

For $\triangle RST$, $x = 9$, and the measure of each side is 18.

CHECK Your Progress

3. Find x and the measure of the unknown sides of isosceles triangle EFG.

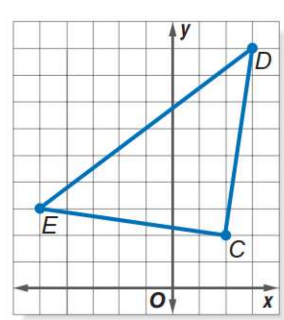


Study Tip

Look Back
To review the **Distance Formula**, see Lesson 1-3.

EXAMPLE Use the Distance Formula

4 **COORDINATE GEOMETRY** **Find the measures of the sides of $\triangle DEC$. Classify the triangle by sides.**



Use the Distance Formula to find the lengths of each side.

$$EC = \sqrt{(-5 - 2)^2 + (3 - 2)^2}$$
$$= \sqrt{49 + 1}$$
$$= \sqrt{50} \text{ or } 5\sqrt{2}$$

$$DC = \sqrt{(3-2)^2 + (9-2)^2} \qquad ED = \sqrt{(-5-3)^2 + (3-9)^2}$$
$$= \sqrt{1 + 49} \qquad = \sqrt{64 + 36}$$
$$= \sqrt{50} \text{ or } 5\sqrt{2} \qquad = \sqrt{100} \text{ or } 10$$

Since $\overline{EC}$ and $\overline{DC}$ have the same length, $\triangle DEC$ is isosceles.

CHECK Your Progress

4. Find the measures of the sides of $\triangle HIJ$ with vertices $H(-3, 1)$, $I(0, 4)$, and $J(0, 1)$. Classify the triangle by sides.

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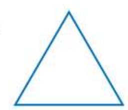
CHECK Your Understanding

Example 1 (p. 203)

Use a protractor to classify each triangle as *acute*, *equiangular*, *obtuse*, or *right*.

1.

2.

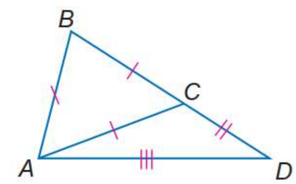


Example 2 (p. 204)

Identify the indicated type of triangle in the figure.

3. isosceles

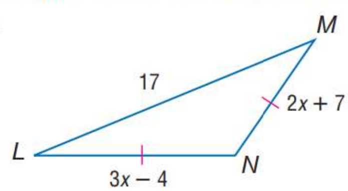
4. scalene



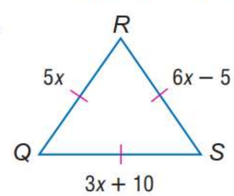
Example 3 (p. 204)

ALGEBRA Find x and the measures of the unknown sides of each triangle.

5.



6.



Example 4 (p. 204)

7. COORDINATE GEOMETRY Find the measures of the sides of $\triangle TWZ$ with vertices at $T(2, 6)$, $W(4, -5)$, and $Z(-3, 0)$. Classify the triangle by sides.

8. COORDINATE GEOMETRY Find the measures of the sides of $\triangle QRS$ with vertices at $Q(2, 1)$, $R(4, -3)$, and $S(-3, -2)$. Classify the triangle by sides.

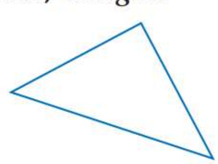
Exercises

HOMEWORK HELP

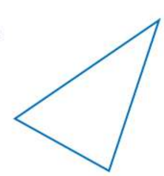
For Exercises	See Examples
9–12	1
13–14	2
15, 16	3
17–20	4

Use a protractor to classify each triangle as *acute*, *equiangular*, *obtuse*, or *right*.

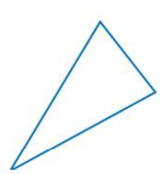
9.



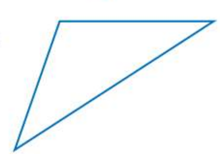
10.



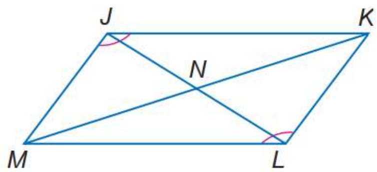
11.



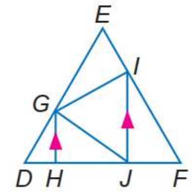
12.



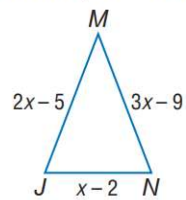
13. Identify the obtuse triangles if $\angle MJK \cong \angle KLM$, $m\angle MJK = 126$, and $m\angle JNM = 52$.



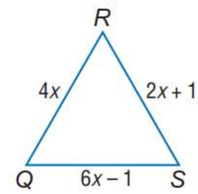
14. Identify the right triangles if $\overline{IJ} \parallel \overline{GH}$, $\overline{GH} \perp \overline{DF}$, and $\overline{GI} \perp \overline{EF}$.



15. ALGEBRA Find x, JM, MN, and JN if $\triangle JMN$ is an isosceles triangle with $\overline{JM} \cong \overline{MN}$.



16 ALGEBRA Find x, QR, RS, and QS if $\triangle QRS$ is an equilateral triangle.



COORDINATE GEOMETRY Find the measures of the sides of $\triangle ABC$ and classify each triangle by its sides.

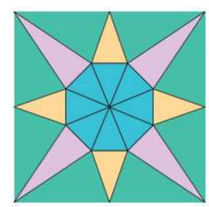
17. $A(5, 4)$, $B(3, -1)$, $C(7, -1)$

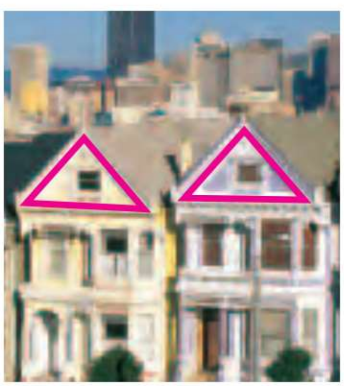
18. $A(-4, 1)$, $B(5, 6)$, $C(-3, -7)$

19. $A(-7, 9)$, $B(-7, -1)$, $C(4, -1)$

20. $A(-3, -1)$, $B(2, 1)$, $C(2, -3)$

21. QUILTING The star-shaped composite quilting square is made up of four different triangles. Use a ruler to classify the four triangles by sides.





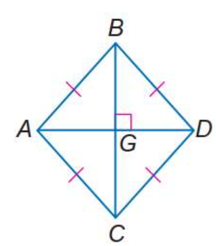
Real-World Link

The Painted Ladies are located in Alamo Square. The area is one of 11 designated historic districts in San Francisco.

Source: www.sfvisitor.org

22. ARCHITECTURE The restored and decorated Victorian houses in San Francisco shown in the photograph are called the "Painted Ladies." Use a protractor to classify the triangles indicated in the photo by sides and angles.

Identify the indicated triangles in the figure if $\overline{AB} \cong \overline{BD} \cong \overline{DC} \cong \overline{CA}$ and $\overline{BC} \perp \overline{AD}$.



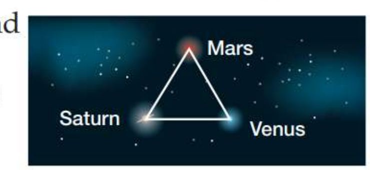
23. right

24. obtuse

25. scalene

26. isosceles

27. ASTRONOMY On May 5, 2002, Venus, Saturn, and Mars were aligned in a triangular formation. Use a protractor or ruler to classify the triangle formed by sides and angles.



28. RESEARCH Use the Internet or other resource to find out how astronomers can predict planetary alignment.

ALGEBRA **Find x and the measure of each side of the triangle.**

29. $\triangle GHJ$ is isosceles, with $\overline{HG} \cong \overline{JG}$, $GH = x + 7$, $GJ = 3x - 5$, and $HJ = x - 1$.

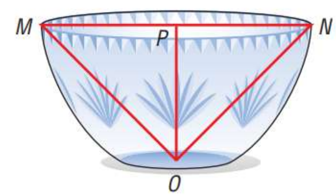
30. $\triangle MPN$ is equilateral with $MN = 3x - 6$, $MP = x + 4$, and $NP = 2x - 1$.

31. $\triangle QRS$ is equilateral. QR is two less than two times a number, RS is six more than the number, and QS is ten less than three times the number.

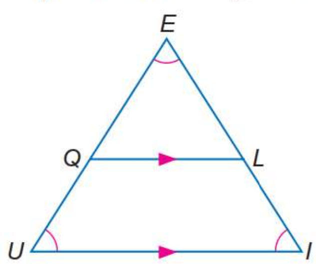
32. $\triangle JKL$ is isosceles with $\overline{KJ} \cong \overline{LJ}$. JL is five less than two times a number. JK is three more than the number. KL is one less than the number. Find the measure of each side.

33. ROAD TRIP The total distance from Charlotte to Raleigh to Winston-Salem and back to Charlotte is about 292 miles. The distance from Charlotte to Winston-Salem is 22 miles less than the distance from Raleigh to Winston-Salem. The distance from Charlotte to Raleigh is 60 miles greater than the distance from Winston-Salem to Charlotte. Classify the triangle that connects Charlotte, Raleigh, and Winston-Salem.

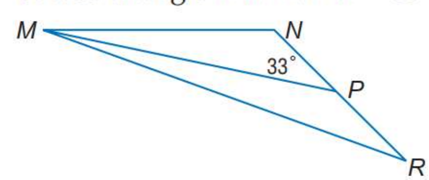
34. CRYSTAL The top of the crystal bowl pictured at the right is circular. The diameter at the top of the bowl is $\overline{MN}$. P is the midpoint of $\overline{MN}$, and $\overline{OP} \perp \overline{MN}$. If $MN = 24$ and $OP = 12$, determine whether $\triangle MPO$ and $\triangle NPO$ are equilateral.



35. PROOF Write a two-column proof to prove that $\triangle EQL$ is equiangular.

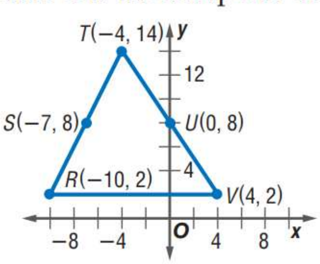


36. PROOF Write a paragraph proof to prove that $\triangle RPM$ is an obtuse triangle if $m\angle NPM = 33$.

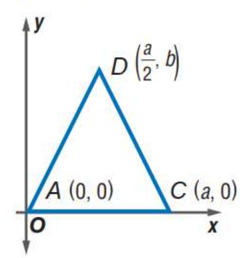


EXTRA PRACTICE
See pages 807, 831.
Math Online
Self-Check Quiz at
ca.geometryonline.com

37. COORDINATE GEOMETRY Show that S is the midpoint of $\overline{RT}$ and U is the midpoint of $\overline{TV}$.



38. COORDINATE GEOMETRY Show that $\triangle ADC$ is isosceles.



H.O.T. Problems

39. OPEN ENDED Draw an isosceles right triangle.

REASONING **Determine whether each statement is *always*, *sometimes*, or *never* true. Explain.**

40. Equiangular triangles are also acute.

41. Right triangles are acute.

42. CHALLENGE $\overline{KL}$ is a segment representing one side of isosceles right triangle KLM with $K(2, 6)$, and $L(4, 2)$. $\angle KLM$ is a right angle, and $\overline{KL} \cong \overline{LM}$. Describe how to find the coordinates of M and name these coordinates.

43. *Writing in Math* Use the information on page 202 to explain why triangles are important in construction. Include a description of how to classify triangles and a justification of why you think one type of triangle might be used more often in architecture than other types.

STANDARDS PRACTICE

44. Which type of triangle can serve as a counterexample to the conjecture below?

If two angles of a triangle are acute, then the measure of the third angle must be greater than or equal to 90.

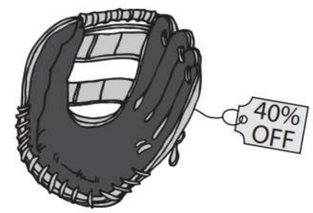
A equilateral

B obtuse

C right

D scalene

45. A baseball glove originally cost \$84.50. Jamal bought it at 40% off.



How much was deducted from the original price?

F \$50.70 **H** \$33.80

G \$44.50 **J** \$32.62

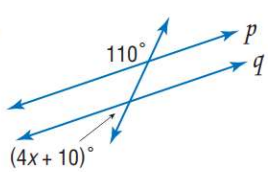
Spiral Review

Graph each line. Construct a perpendicular segment through the given point. Then find the distance from the point to the line. (Lesson 3-6)

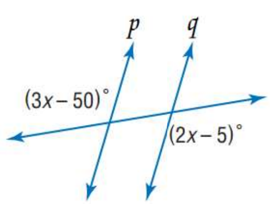
46. $y = x + 2$, $(2, -2)$ **47.** $x + y = 2$, $(3, 3)$ **48.** $y = 7$, $(6, -2)$

Find x so that $p \parallel q$. (Lesson 3-5)

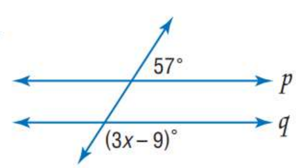
49.



50.



51.



GET READY for the Next Lesson

PREREQUISITE SKILL In the figure, $\overline{AB} \parallel \overline{RQ}$, $\overline{BC} \parallel \overline{PR}$, and $\overline{AC} \parallel \overline{PQ}$. Name the indicated angles or pairs of angles. (Lessons 3-1 and 3-2)

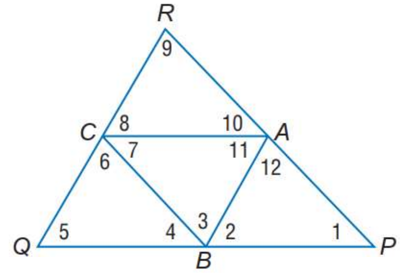
52. three pairs of alternate interior angles

53. six pairs of corresponding angles

54. all angles congruent to $\angle 3$

55. all angles congruent to $\angle 7$

56. all angles congruent to $\angle 11$



EXPLORE 4-2

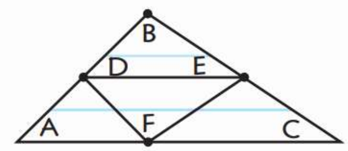
Geometry Lab

Angles of Triangles

Standard 13.0 Students prove relationships between angles in polygons by using properties of complementary, **supplementary,** vertical, and **exterior angles.**

ACTIVITY 1

Find the relationship among the measures of the interior angles of a triangle.



Step 1 Draw an obtuse triangle and cut it out. Label the vertices A, B, and C.

Step 2 Find the midpoint of $\overline{AB}$ by matching A to B. Label this point D.

Step 3 Find the midpoint of $\overline{BC}$ by matching B to C. Label this point E.

Step 4 Draw $\overline{DE}$.

Step 5 Fold $\triangle ABC$ along $\overline{DE}$. Label the point where B touches $\overline{AC}$ as F.

Step 6 Draw $\overline{DF}$ and $\overline{FE}$. Measure each angle.

ANALYZE THE MODEL

Describe the relationship between each pair.

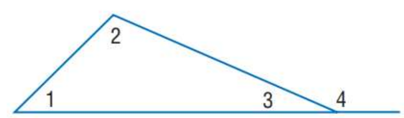
1. $\angle A$ and $\angle DFA$ **2.** $\angle B$ and $\angle DFE$ **3.** $\angle C$ and $\angle EFC$

4. What is the sum of the measures of $\angle DFA$, $\angle DFE$, and $\angle EFC$?

5. What is the sum of the measures of $\angle A$, $\angle B$, and $\angle C$?

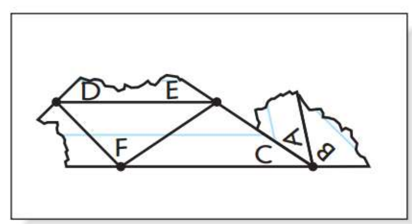
6. Make a conjecture about the sum of the measures of the angles of any triangle.

In the figure at the right, $\angle 4$ is called an *exterior angle* of the triangle. $\angle 1$ and $\angle 2$ are the *remote interior angles* of $\angle 4$.



ACTIVITY 2

Find the relationship among the interior and exterior angles of a triangle.



Step 1 Trace $\triangle ABC$ from Activity 1 onto a piece of paper. Label the vertices.

Step 2 Extend $\overline{AC}$ to draw an exterior angle at C.

Step 3 Tear $\angle A$ and $\angle B$ off the triangle from Activity 1.

Step 4 Place $\angle A$ and $\angle B$ over the exterior angle.

ANALYZE THE RESULTS

7. Make a conjecture about the relationship of $\angle A$, $\angle B$, and the exterior angle at C.

8. Repeat the steps for the exterior angles of $\angle A$ and $\angle B$.

9. Is your conjecture true for all exterior angles of a triangle?

10. Repeat Activity 2 with an acute triangle and with a right triangle.

11. Make a conjecture about the measure of an exterior angle and the sum of the measures of its remote interior angles.

4-2 Angles of Triangles

Main Ideas

- Apply the Angle Sum Theorem.
- Apply the Exterior Angle Theorem.

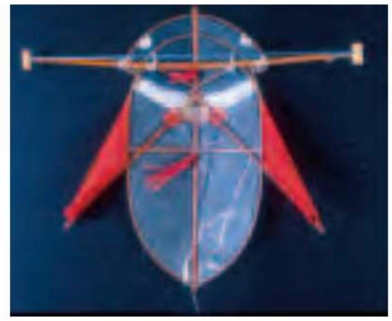
Standard 13.0 Students prove relationships between angles in polygons by using properties of complementary, supplementary, vertical, and exterior angles.

New Vocabulary

exterior angle
remote interior angles
flow proof
corollary

GET READY for the Lesson

The Drachen Foundation coordinates the annual Miniature Kite Contest. In a recent year, the kite in the photograph won second place in the Most Beautiful Kite category. The overall dimensions are 10.5 centimeters by 9.5 centimeters. The wings of the beetle are triangular.

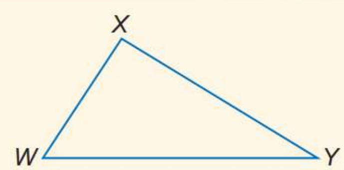


Angle Sum Theorem If the measures of two of the angles of a triangle are known, how can the measure of the third angle be determined? The Angle Sum Theorem explains that the sum of the measures of the angles of any triangle is always 180.

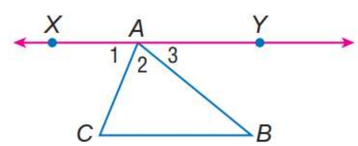
THEOREM 4.1 *Angle Sum*

The sum of the measures of the angles of a triangle is 180.

Example: $m\angle W + m\angle X + m\angle Y = 180$



PROOF Angle Sum Theorem



Given: $\triangle ABC$

Prove: $m\angle C + m\angle 2 + m\angle B = 180$

Proof:

Statements	Reasons
1. $\triangle ABC$	1. Given
2. Draw $\overleftrightarrow{XY}$ through A parallel to $\overline{CB}$.	2. Parallel Postulate
3. $\angle 1$ and $\angle CAY$ form a linear pair.	3. Def. of a linear pair
4. $\angle 1$ and $\angle CAY$ are supplementary.	4. If 2 ∠s form a linear pair, they are supplementary.
5. $m\angle 1 + m\angle CAY = 180$	5. Def. of suppl. ∠s
6. $m\angle CAY = m\angle 2 + m\angle 3$	6. Angle Addition Postulate
7. $m\angle 1 + m\angle 2 + m\angle 3 = 180$	7. Substitution
8. $\angle 1 \cong \angle C$, $\angle 3 \cong \angle B$	8. Alt. Int. ∠s Theorem
9. $m\angle 1 = m\angle C$, $m\angle 3 = m\angle B$	9. Def. of ≅ ∠s
10. $m\angle C + m\angle 2 + m\angle B = 180$	10. Substitution

Study Tip

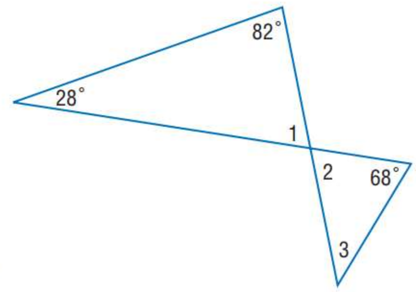
Auxiliary Lines

Recall that sometimes extra lines have to be drawn to complete a proof. These are called *auxiliary lines*.

If we know the measures of two angles of a triangle, we can find the measure of the third.

EXAMPLE Interior Angles

1 Find the missing angle measures.



Find $m\angle 1$ first because the measures of two angles of the triangle are known.

$m\angle 1 + 28 + 82 = 180$ Angle Sum Theorem

$m\angle 1 + 110 = 180$ Simplify.

$m\angle 1 = 70$ Subtract 110 from each side.

$\angle 1$ and $\angle 2$ are congruent vertical angles. So $m\angle 2 = 70$.

$m\angle 3 + 68 + 70 = 180$ Angle Sum Theorem

$m\angle 3 + 138 = 180$ Simplify.

$m\angle 3 = 42$ Subtract 138 from each side.

Therefore, $m\angle 1 = 70$, $m\angle 2 = 70$, and $m\angle 3 = 42$.

Study Tip

Mental Math

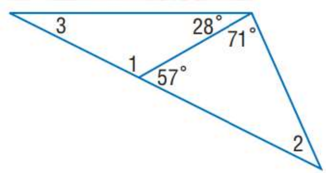
You can also use mental math to solve the equation $m\angle 3 + 138 = 180$.

Think: $138 + 2 = 140$ and $140 + 40 = 180$.

So, $m\angle 3 = 2 + 40$ or 42.

CHECK Your Progress

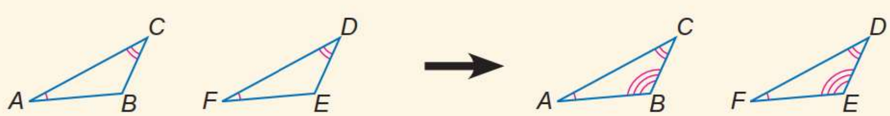
1.



The Angle Sum Theorem leads to a useful theorem about the angles in two triangles.

THEOREM 4.2 ***Third Angle Theorem***

If two angles of one triangle are congruent to two angles of a second triangle, then the third angles of the triangles are congruent.



Example: If $\angle A \cong \angle F$ and $\angle C \cong \angle D$, then $\angle B \cong \angle E$.

You will prove this theorem in Exercise 34.

Vocabulary Link

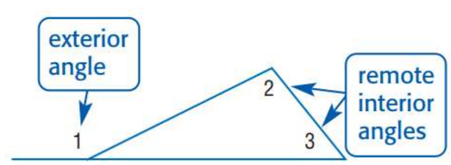
Remote

Everyday Use located far away; distant in space

Interior

Everyday Use the internal portion or area

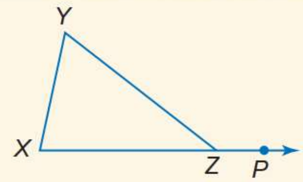
Exterior Angle Theorem Each angle of a triangle has an exterior angle. An **exterior angle** is formed by one side of a triangle and the extension of another side. The interior angles of the triangle not adjacent to a given exterior angle are called **remote interior angles** of the exterior angle.



THEOREM 4.3 — *Exterior Angle Theorem*

The measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles.

Example: $m\angle X + m\angle Y = m\angle YZP$



Study Tip

Flow Proof
Write each statement and reason on an index card. Then organize the index cards in logical order.

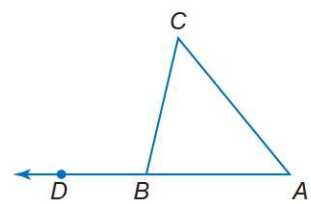
We will use a flow proof to prove this theorem. A **flow proof** organizes a series of statements in logical order, starting with the given statements. Each statement is written in a box with the reason verifying the statement written below the box. Arrows are used to indicate how the statements relate to each other.

PROOF — Exterior Angle Theorem

Write a flow proof of the Exterior Angle Theorem.

Given: $\triangle ABC$

Prove: $m\angle CBD = m\angle A + m\angle C$



Flow Proof:

$\triangle ABC$
Given

$\angle CBD$ and $\angle ABC$ form a linear pair.
Definition of linear pair

$\angle CBD$ and $\angle ABC$ are supplementary.
If 2 ∠s form a linear pair, they are supplementary.

$m\angle A + m\angle ABC + \angle C = 180$
Angle Sum Theorem

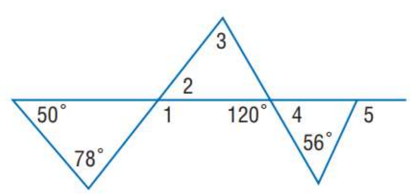
$m\angle CBD + m\angle ABC = 180$
Definition of supplementary

$m\angle A + m\angle ABC + m\angle C = m\angle CBD + m\angle ABC$
Substitution Property

$m\angle A + m\angle C = m\angle CBD$
Subtraction Property

EXAMPLE — Exterior Angles

2 Find the measure of each angle.



a. $m\angle 1$

$m\angle 1 = 50 + 78$ Exterior Angle Theorem

$= 128$ Simplify.

b. $m\angle 2$

$m\angle 1 + m\angle 2 = 180$ If 2∠s form a linear pair, they are suppl.

$128 + m\angle 2 = 180$ Substitution

$m\angle 2 = 52$ Subtract 128 from each side.

c. $m\angle 3$

$m\angle 2 + m\angle 3 = 120$ — Exterior Angle Theorem

$52 + m\angle 3 = 120$ — Substitution

$m\angle 3 = 68$ — Subtract 52 from each side.

Therefore, $m\angle 1 = 128$, $m\angle 2 = 52$, and $m\angle 3 = 68$.

CHECK Your Progress

2A. $m\angle 4$

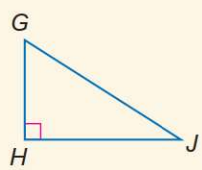
2B. $m\angle 5$

Personal Tutor at ca.geometryonline.com

A statement that can be easily proved using a theorem is often called a **corollary** of that theorem. A corollary, just like a theorem, can be used as a reason in a proof.

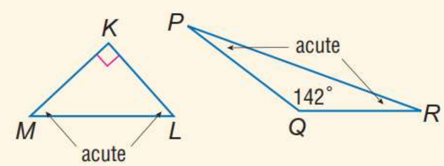
COROLLARIES

4.1 The acute angles of a right triangle are complementary.



Example: $m\angle G + m\angle J = 90$

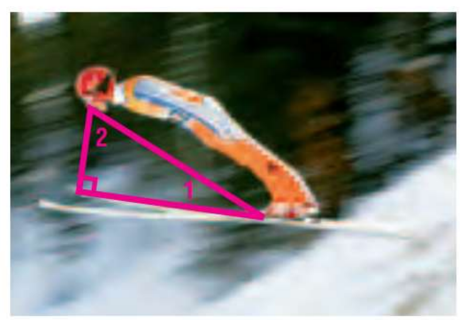
4.2 There can be at most one right or obtuse angle in a triangle.



You will prove Corollaries 4.1 and 4.2 in Exercises 32 and 33.

Real-World EXAMPLE Right Angles

3 **SKI JUMPING** **Ski jumper Simon Ammann of Switzerland forms a right triangle with his skis and his line of sight. Find $m\angle 2$ if $m\angle 1$ is 27.**



Use Corollary 4.1 to write an equation.

$m\angle 1 + m\angle 2 = 90$

$27 + m\angle 2 = 90$ — Substitution

$m\angle 2 = 63$ — Subtract 27 from each side.

CHECK Your Progress

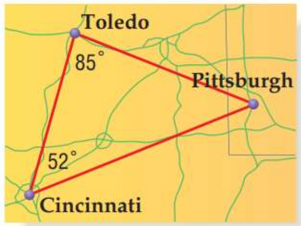
3. WIND SURFING A windsurfing sail is generally a right triangle. One of the angles that is not the right angle has a measure of 68°. What is the measure of the other nonright angle?

CHECK Your Understanding

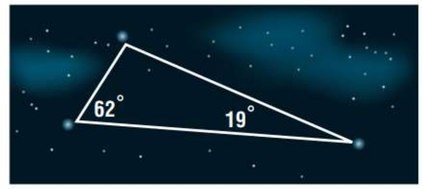
Example 1 (p. 211)

Find the missing angle measure.

1.



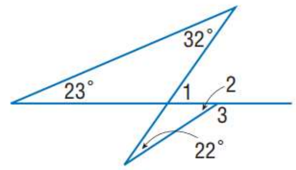
2.



Example 2 (p. 212)

Find each measure.

3. $m\angle 1$ **4.** $m\angle 2$ **5.** $m\angle 3$

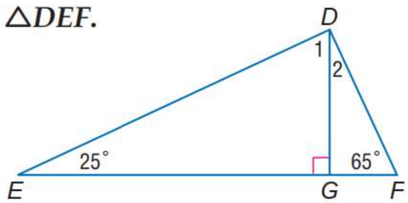


Example 3 (p. 213)

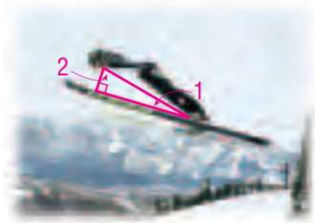
Find each measure in $\triangle DEF$.

6. $m\angle 1$

7. $m\angle 2$



8. SKI JUMPING American ski jumper Jessica Jerome forms a right angle with her skis. If $m\angle 2 = 70$, find $m\angle 1$.

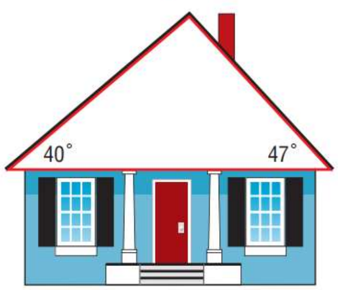


Exercises

HOMEWORK HELP	
For Exercises	See Examples
9–12	1
13–18	2
19–22	3

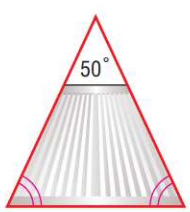
Find the missing angle measures.

9.

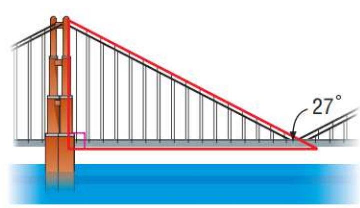


10.

11.



12.

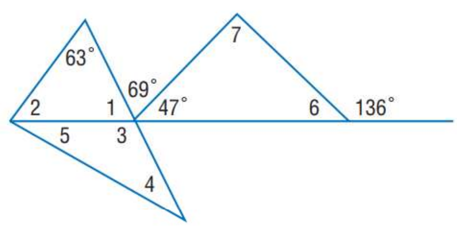


Find each measure if $m\angle 4 = m\angle 5$.

13. $m\angle 1$ **14.** $m\angle 2$

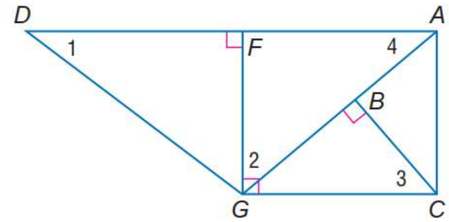
15. $m\angle 3$ **16.** $m\angle 4$

17. $m\angle 5$ **18.** $m\angle 6$



Doug Pensinger/Getty Images

Find each measure if $m\angle DGF = 53$ and $m\angle AGC = 40$.



19. $m\angle 1$

20. $m\angle 2$

21. $m\angle 3$

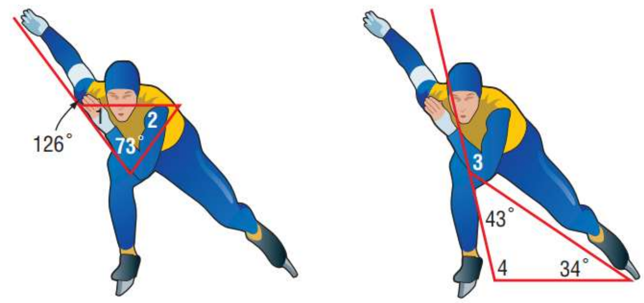
22. $m\angle 4$

Real-World Link

Catriona Lemay Doan is the first Canadian to win a Gold medal in the same event in two consecutive Olympic games.

Source: catrionalemaydoan.com

SPEED SKATING **For Exercises 23–26, use the following information.** Speed skater Catriona Lemay Doan of Canada forms at least two sets of triangles and exterior angles as she skates. Use the measures of given angles to find each measure.



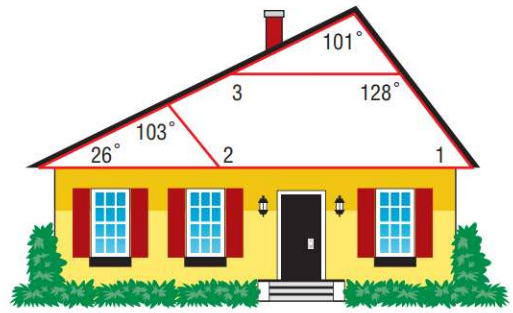
23. $m\angle 1$

24. $m\angle 2$

25. $m\angle 3$

26. $m\angle 4$

HOUSING **For Exercises 27–29, use the following information.** The two braces for the roof of a house form triangles. Find each measure.



27. $m\angle 1$

28. $m\angle 2$

29. $m\angle 3$

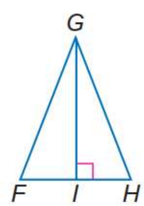
PROOF **For Exercises 30–34, write the specified type of proof.**

30. flow proof

Given: $\angle FGI \cong \angle IGH$

$\overline{GI} \perp \overline{FH}$

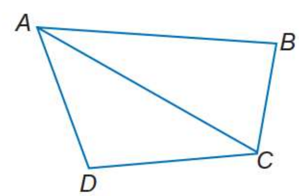
Prove: $\angle F \cong \angle H$



31. two-column proof

Given: $ABCD$ is a quadrilateral.

Prove: $m\angle DAB + m\angle B + m\angle BCD + m\angle D = 360$



32. flow proof of Corollary 4.1

33. paragraph proof of Corollary 4.2

34. two-column proof of Theorem 4.2

EXTRA PRACTICE

See pages 807, 831.

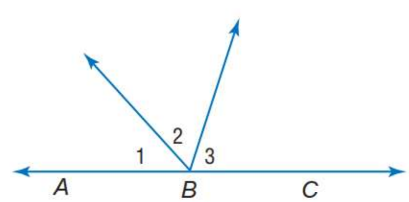
Math Online

Self-Check Quiz at ca.geometryonline.com

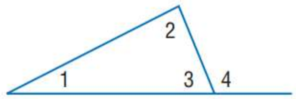
H.O.T. Problems

35. OPEN ENDED Draw a triangle. Label one exterior angle and its remote interior angles.

36. CHALLENGE $\overrightarrow{BA}$ and $\overrightarrow{BC}$ are opposite rays. The measures of $\angle 1$, $\angle 2$, and $\angle 3$ are in a 4:5:6 ratio. Find the measure of each angle.



37. FIND THE ERROR Najee and Kara are discussing the Exterior Angle Theorem. Who is correct? Explain.



Najee	Kara
$m\angle 1 + m\angle 2 = m\angle 4$	$m\angle 1 + m\angle 2 + m\angle 4 = 180$

38. *Writing in Math* Use the information about kites provided on page 210 to explain how the angles of triangles are used to make kites. Include an explanation of how you can find the measure of a third angle if two angles of two triangles are congruent. Also include a description of the properties of two angles in a triangle if the measure of the third is 90°.

STANDARDS PRACTICE

39. Two angles of a triangle have measures of 35° and 80°. Which of the following could *not* be a measure of an exterior angle of the triangle?

A 165°

B 145°

C 115°

D 100°

40. Which equation is equivalent to $7x - 3(2 - 5x) = 8x$?

F $2x - 6 = 8x$

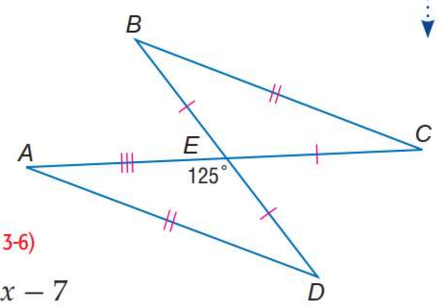
G $22x - 6 = 8x$

H $-8x - 6 = 8x$

J $22x + 6 = 8x$

Spiral Review

Identify the indicated triangles if $\overline{BC} \cong \overline{AD}$, $\overline{EB} \cong \overline{EC}$, $\overline{AC}$ bisects $\overline{BD}$, and $m\angle AED = 125$. (Lesson 4-1)



41. scalene

42. obtuse

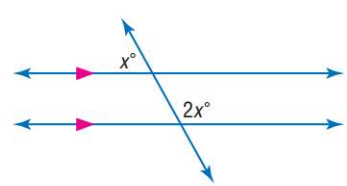
43. isosceles

Find the distance between each pair of parallel lines. (Lesson 3-6)

44. $y = x + 6, y = x - 10$

45. $y = -2x + 3, y = -2x - 7$

46. MODEL TRAINS Regan is going to set up two parallel train tracks with a third track running diagonally across the first two. To properly place a switch, she needs the angle between the diagonal and top of the second track to be twice as large as the angle between the diagonal and top of the first track. What is the value of x? (Lesson 3-2)



GET READY for the Next Lesson

PREREQUISITE SKILL List the property of congruence used for each statement. (Lessons 2-5 and 2-6)

47. $\angle 1 \cong \angle 1$ and $\overline{AB} \cong \overline{AB}$.

48. If $\overline{AB} \cong \overline{XY}$, then $\overline{XY} \cong \overline{AB}$.

49. If $\angle 1 \cong \angle 2$, then $\angle 2 \cong \angle 1$.

50. If $\angle 2 \cong \angle 3$ and $\angle 3 \cong \angle 4$, then $\angle 2 \cong \angle 4$.

4-3 Congruent Triangles

Main Ideas

- Name and label corresponding parts of congruent triangles.
- Identify congruence transformations.

Standard 5.0 **Students prove that triangles are congruent** or similar, **and they are able to use the concept of corresponding parts of congruent triangles.**

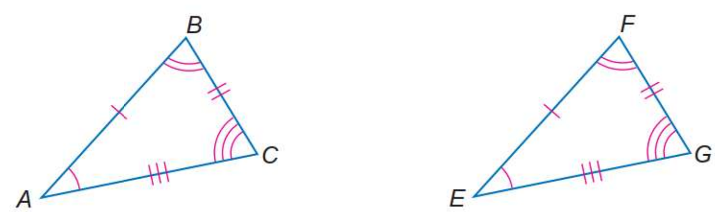
New Vocabulary

congruent triangles
congruence transformations

GET READY for the Lesson

The western portion of the San Francisco-Oakland Bay Bridge spans almost 1.8 miles from San Francisco to Yerba Buena Island. Steel beams, arranged along the side in a triangular web, add structure and stability to the bridge. Triangles spread weight and stress evenly throughout of the bridge.

Corresponding Parts of Congruent Triangles Triangles that are the same size and shape are **congruent triangles**. Each triangle has three angles and three sides. If all six of the corresponding parts of two triangles are congruent, then the triangles are congruent.



If $\triangle ABC$ is congruent to $\triangle EFG$, the vertices of the two triangles correspond in the same order as the letters naming the triangles.

$$\triangle ABC \cong \triangle EFG$$

This correspondence of vertices can be used to name the corresponding congruent sides and angles of the two triangles.

$$\angle A \cong \angle E \qquad \angle B \cong \angle F \qquad \angle C \cong \angle G$$

$$\overline{AB} \cong \overline{EF} \qquad \overline{BC} \cong \overline{FG} \qquad \overline{AC} \cong \overline{EG}$$

The corresponding sides and angles can be determined from any congruence statement by following the order of the letters.

Study Tip

Congruent Parts
In congruent triangles, congruent sides are opposite congruent angles.

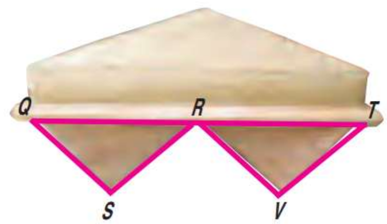
KEY CONCEPT ***Definition of Congruent Triangles (CPCTC)***

Two triangles are congruent if and only if their corresponding parts are congruent.

CPCTC stands for *corresponding parts of congruent triangles are congruent*. "If and only if" is used to show that both the conditional and its converse are true.

Real-World EXAMPLE **Corresponding Congruent Parts**

1 **FURNITURE DESIGN** **The legs of this stool form two triangles. Suppose the measures in inches are $QR = 12$, $RS = 23$, $QS = 24$, $RT = 12$, $TV = 24$, and $RV = 23$.**



a. **Name the corresponding congruent angles and sides.**

$\angle Q \cong \angle T$	$\angle QRS \cong \angle TRV$	$\angle S \cong \angle V$
$\overline{QR} \cong \overline{TR}$	$\overline{RS} \cong \overline{RV}$	$\overline{QS} \cong \overline{TV}$

b. **Name the congruent triangles.**

$\triangle QRS \cong \triangle TRV$

CHECK Your Progress

The measures of the sides of triangles PDQ and OEC are $PD = 5$, $DQ = 7$, $PQ = 11$; $EC = 7$, $OC = 5$, and $OE = 11$.

1A. Name the corresponding congruent angles and sides.

1B. Name the congruent triangles.

Like congruence of segments and angles, congruence of triangles is reflexive, symmetric, and transitive.

THEOREM 4.4 *Properties of Triangle Congruence*

Congruence of triangles is reflexive, symmetric, and transitive.

Reflexive

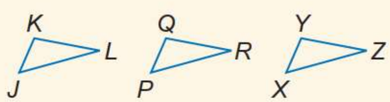
$\triangle JKL \cong \triangle JKL$

Symmetric

If $\triangle JKL \cong \triangle PQR$, then $\triangle PQR \cong \triangle JKL$.

Transitive

If $\triangle JKL \cong \triangle PQR$, and $\triangle PQR \cong \triangle XYZ$, then $\triangle JKL \cong \triangle XYZ$.



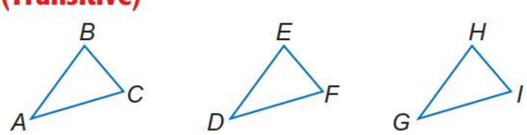
You will prove the symmetric and reflexive parts of Theorem 4.4 in Exercises 30 and 32, respectively.

Proof **Theorem 4.4 (Transitive)**

Given: $\triangle ABC \cong \triangle DEF$

$\triangle DEF \cong \triangle GHI$

Prove: $\triangle ABC \cong \triangle GHI$



Proof: You are given that $\triangle ABC \cong \triangle DEF$. Because corresponding parts of congruent triangles are congruent, $\angle A \cong \angle D$, $\angle B \cong \angle E$, $\angle C \cong \angle F$, $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, and $\overline{AC} \cong \overline{DF}$. You are also given that $\triangle DEF \cong \triangle GHI$. So $\angle D \cong \angle G$, $\angle E \cong \angle H$, $\angle F \cong \angle I$, $\overline{DE} \cong \overline{GH}$, $\overline{EF} \cong \overline{HI}$, and $\overline{DF} \cong \overline{GI}$, by CPCTC. Therefore, $\angle A \cong \angle G$, $\angle B \cong \angle H$, $\angle C \cong \angle I$, $\overline{AB} \cong \overline{GH}$, $\overline{BC} \cong \overline{HI}$, and $\overline{AC} \cong \overline{GI}$ because congruence of angles and segments is transitive. Thus, $\triangle ABC \cong \triangle GHI$ by the definition of congruent triangles.

Study Tip

Naming Congruent Triangles

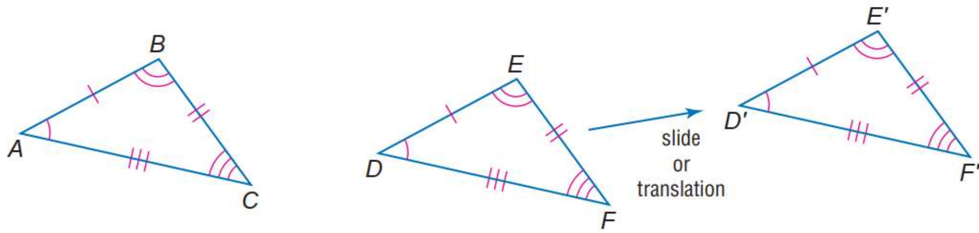
There are six ways to name each pair of congruent triangles.

Study Tip

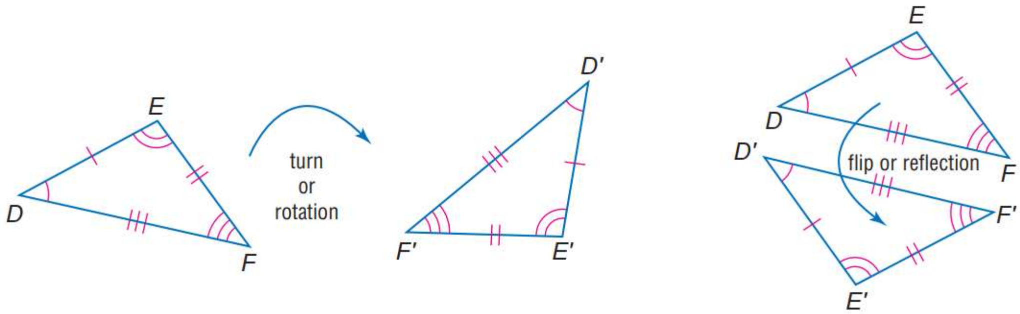
Transformations

Not all transformations preserve congruence. Only transformations that do not change the size or shape of the figure are congruence transformations. You will learn more about transformations in Chapter 9.

Identify Congruence Transformations In the figures below, $\triangle ABC$ is congruent to $\triangle DEF$. If you *slide*, or *translate*, $\triangle DEF$ up and to the right, $\triangle DEF$ is still congruent to $\triangle ABC$.



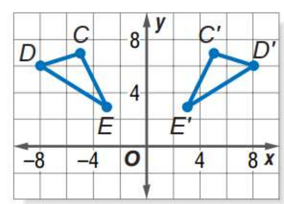
The congruency does not change whether you *turn*, or *rotate*, $\triangle DEF$ or *flip*, or *reflect*, $\triangle DEF$. $\triangle ABC$ is still congruent to $\triangle DEF$.



If you slide, flip, or turn a triangle, the size and shape do not change. These three transformations are called **congruence transformations**.

EXAMPLE Transformations in the Coordinate Plane

2 **COORDINATE GEOMETRY** **The vertices of $\triangle CDE$ are $C(-5, 7)$, $D(-8, 6)$, and $E(-3, 3)$. The vertices of $\triangle C'D'E'$ are $C'(5, 7)$, $D'(8, 6)$, and $E'(3, 3)$.**



a. Verify that $\triangle CDE \cong \triangle C'D'E'$.

Use the Distance Formula to find the length of each side in the triangles.

$$DC = \sqrt{[-8-(-5)]^2 + (6-7)^2} = \sqrt{9+1} \text{ or } \sqrt{10}$$

$$D'C' = \sqrt{(8-5)^2 + (6-7)^2} = \sqrt{9+1} \text{ or } \sqrt{10}$$

$$DE = \sqrt{[-8-(-3)]^2 + (6-3)^2} = \sqrt{25+9} \text{ or } \sqrt{34}$$

$$D'E' = \sqrt{(8-3)^2 + (6-3)^2} = \sqrt{25+9} \text{ or } \sqrt{34}$$

$$CE = \sqrt{[-5-(-3)]^2 + (7-3)^2} = \sqrt{4+16} = \sqrt{20} \text{ or } 2\sqrt{5}$$

$$C'E' = \sqrt{(5-3)^2 + (7-3)^2} = \sqrt{4+16} = \sqrt{20} \text{ or } 2\sqrt{5}$$

By the definition of congruence, $\overline{DC} \cong \overline{D'C'}$, $\overline{DE} \cong \overline{D'E'}$, and $\overline{CE} \cong \overline{C'E'}$. Use a protractor to measure the angles of the triangles. You will find that the measures are the same.

In conclusion, because $\overline{DC} \cong \overline{D'C'}$, $\overline{DE} \cong \overline{D'E'}$, and $\overline{CE} \cong \overline{C'E'}$, $\angle D \cong \angle D'$, $\angle C \cong \angle C'$, and $\angle E \cong \angle E'$, $\triangle CDE \cong \triangle C'D'E'$.

(continued on the next page)

b. Name the congruence transformation for $\triangle CDE$ and $\triangle C'D'E'$.

$\triangle C'D'E'$ is a flip, or reflection, of $\triangle CDE$.

CHECK Your Progress

COORDINATE GEOMETRY The vertices of $\triangle LMN$ are $L(1, 1)$, $M(3, 5)$, and $N(5, 1)$. The vertices of $\triangle L'M'N'$ are $L'(-1, -1)$, $M'(-3, -5)$, and $N'(-5, -1)$.

2A. Verify that $\triangle LMN \cong L'M'N'$.

2B. Name the congruence transformation for $\triangle LMN$ and $\triangle L'M'N'$.

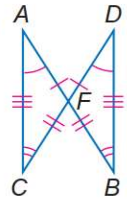
Online **Personal Tutor at** ca.geometryonline.com

CHECK Your Understanding

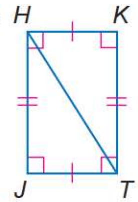
Example 1 (p. 218)

Identify the corresponding congruent angles and sides and the congruent triangles in each figure.

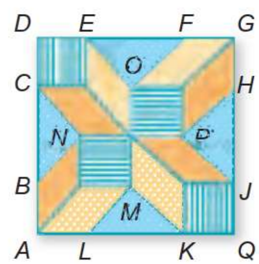
1.



2.



3. QUILTING In the quilt design, assume that angles and segments that appear to be congruent are congruent. Indicate which triangles are congruent.



Example 2 (p. 219)

4. The vertices of $\triangle SUV$ and $\triangle S'U'V'$ are $S(0, 4)$, $U(0, 0)$, $V(2, 2)$, $S'(0, -4)$, $U'(0, 0)$, and $V'(-2, -2)$. Verify that the triangles are congruent and then name the congruence transformation.

5. The vertices of $\triangle QRT$ and $\triangle Q'R'T'$ are $Q(-4, 3)$, $Q'(4, 3)$, $R(-4, -2)$, $R'(4, -2)$, $T(-1, -2)$, and $T'(1, -2)$. Verify that $\triangle QRT \cong \triangle Q'R'T'$. Then name the congruence transformation.

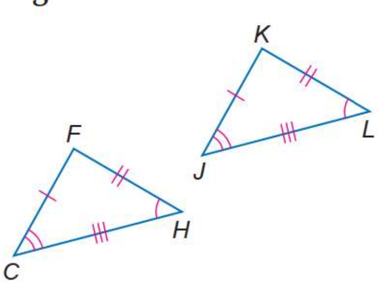
Exercises

HOMEWORK HELP

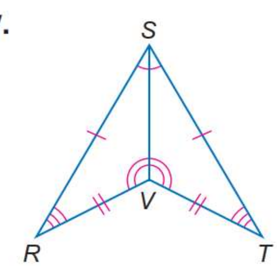
For Exercises	See Examples
6–9	1
10–13	2

Identify the congruent angles and sides and the congruent triangles in each figure.

6.

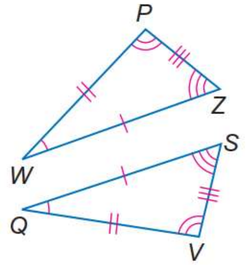


7.

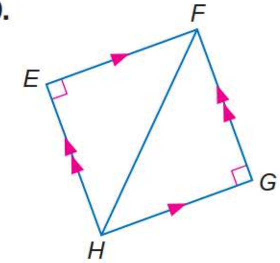


Identify the congruent angles and sides and the congruent triangles in each figure.

8.

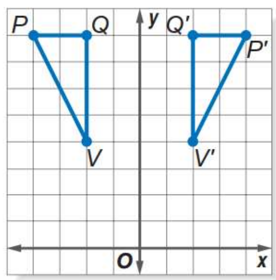


9.

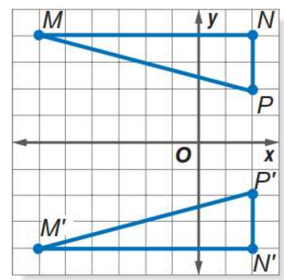


Verify each congruence and name the congruence transformation.

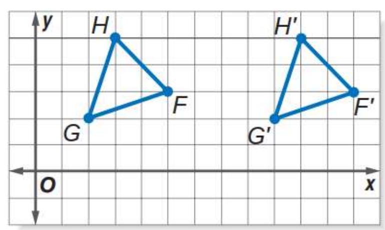
10. $\triangle PQV \cong \triangle P'Q'V'$



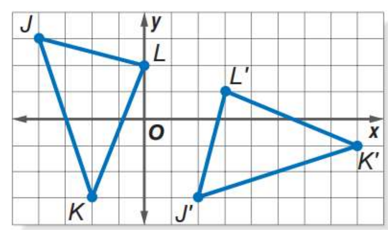
11. $\triangle MNP \cong \triangle M'N'P'$



12. $\triangle GHF \cong \triangle G'H'F'$



13. $\triangle JKL \cong \triangle J'K'L'$



Name the congruent angles and sides for each pair of congruent triangles.

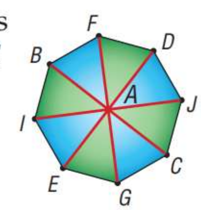
14. $\triangle TUV \cong \triangle XYZ$

15. $\triangle CDG \cong \triangle RSW$

16. $\triangle BCF \cong \triangle DGH$

17. $\triangle ADG \cong \triangle HKL$

18. UMBRELLAS Umbrellas usually have eight triangular sections with ribs of equal length. Are the statements $\triangle JAD \cong \triangle IAE$ and $\triangle JAD \cong \triangle EAI$ both correct? Explain.



Real-World Link

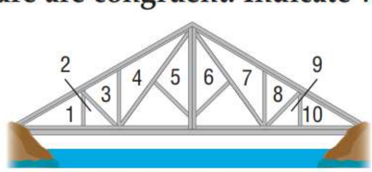
A mosaic is composed of glass, marble, or ceramic pieces often arranged in a pattern. The pieces, or *tesserae*, are set in cement. Mosaics are used to decorate walls, floors, and gardens.

Source: www.dimosaic.com

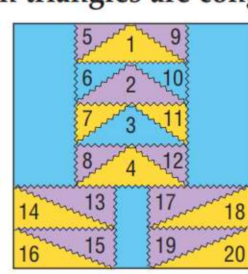
19. MOSAICS The figure at the left is the center of a Roman mosaic. If the bases of the triangles are each the same length, what else do you need to know to conclude that the four triangles surrounding the square are congruent?

Assume that segments and angles that appear to be congruent in each figure are congruent. Indicate which triangles are congruent.

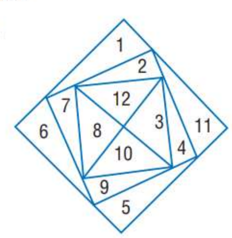
20.



21.



22.



Determine whether each statement is *true* or *false*. Draw an example or counterexample for each.

23. Two triangles with corresponding congruent angles are congruent.

24. Two triangles with angles and sides congruent are congruent.

ALGEBRA For Exercises 25 and 26, use the following information.
$\triangle QRS \cong \triangle GHJ$, $RS = 12$, $QR = 10$, $QS = 6$, and $HJ = 2x - 4$.

25. Draw and label a figure to show the congruent triangles.

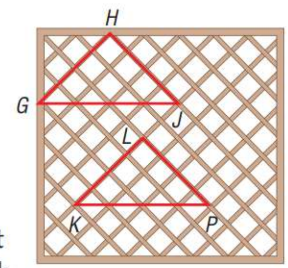
26. Find x.

ALGEBRA For Exercises 27 and 28, use the following information.
$\triangle JKL \cong \triangle DEF$, $m\angle J = 36$, $m\angle E = 64$, and $m\angle F = 3x + 52$.

27. Draw and label a figure to show the congruent triangles.

28. Find x.

29. GARDENING This garden lattice will be covered with morning glories in the summer. Malina wants to save two triangular areas for artwork. If $\triangle GHJ \cong \triangle KLP$, name the corresponding congruent angles and sides.



30. PROOF Put the statements used to prove the statement below in the correct order. Provide the reasons for each statement.

Congruence of triangles is symmetric.

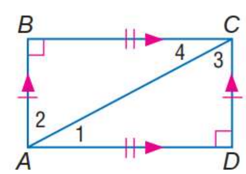
Given: $\triangle RST \cong \triangle XYZ$

Prove: $\triangle XYZ \cong \triangle RST$

Proof:

$\angle X \cong \angle R$, $\angle Y \cong \angle S$, $\angle Z \cong \angle T$, $\overline{XY} \cong \overline{RS}$, $\overline{YZ} \cong \overline{ST}$, $\overline{XZ} \cong \overline{RT}$	$\angle R \cong \angle X$, $\angle S \cong \angle Y$, $\angle T \cong \angle Z$, $\overline{RS} \cong \overline{XY}$, $\overline{ST} \cong \overline{YZ}$, $\overline{RT} \cong \overline{XZ}$	$\triangle RST \cong \triangle XYZ$	$\triangle XYZ \cong \triangle RST$
?	?	?	?

31. PROOF Copy the flow proof and provide the reasons for each statement.



Given: $\overline{AB} \cong \overline{CD}$, $\overline{AD} \cong \overline{CB}$, $\overline{AD} \perp \overline{DC}$, $\overline{AB} \perp \overline{BC}$, $\overline{AD} \parallel \overline{BC}$, $\overline{AB} \parallel \overline{CD}$

Prove: $\triangle ACD \cong \triangle CAB$

Proof:

- $\overline{AB} \cong \overline{CD}$ a. ? → $\triangle ACD \cong \triangle CAB$
- $\overline{AD} \cong \overline{CB}$ b. ? → $\triangle ACD \cong \triangle CAB$
- $\overline{AC} \cong \overline{CA}$ c. ? → $\triangle ACD \cong \triangle CAB$
- $\overline{AD} \perp \overline{DC}$ d. ? → $\angle D$ is a rt. $\angle$. e. ? → $\angle D \cong \angle B$
- $\overline{AB} \perp \overline{BC}$ f. ? → $\angle B$ is a rt. $\angle$. g. ? → $\angle D \cong \angle B$
- $\angle D \cong \angle B$ h. ? → $\triangle ACD \cong \triangle CAB$
- $\overline{AD} \parallel \overline{BC}$ i. ? → $\angle 1 \cong \angle 4$ j. ? → $\triangle ACD \cong \triangle CAB$
- $\overline{AB} \parallel \overline{CD}$ k. ? → $\angle 2 \cong \angle 3$ l. ? → $\triangle ACD \cong \triangle CAB$
- $\triangle ACD \cong \triangle CAB$ m. ?

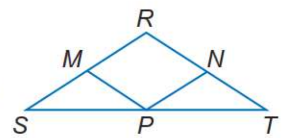
EXTRA PRACTICE
See pages 807, 831.
Math Online
Self-Check Quiz at
ca.geometryonline.com

32. **PROOF** Write a flow proof to prove that congruence of triangles is reflexive. (Theorem 4.4)

H.O.T. Problems

33. **OPEN ENDED** Find a real-world picture of congruent triangles and explain how you know that the triangles are congruent.

34. **CHALLENGE** $\triangle RST$ is isosceles with $RS = RT$, M, N, and P are midpoints of the respective sides, $\angle S \cong \angle MPS$, and $\overline{NP} \cong \overline{MP}$. What else do you need to know to prove that $\triangle SMP \cong \triangle TNP$?



35. *Writing in Math* Use the information on page 217 to explain why triangles are used in the design and construction of bridges.

STANDARDS PRACTICE

36. Triangle ABC is congruent to $\triangle HIJ$. The vertices of $\triangle ABC$ are $A(-1, 2)$, $B(0, 3)$, and $C(2, -2)$. What is the measure of side $\overline{HJ}$?

A $\sqrt{2}$ **C** 5

B 3 **D** cannot be determined

37. **REVIEW** Which is a factor of $x^2 + 19x - 42$?

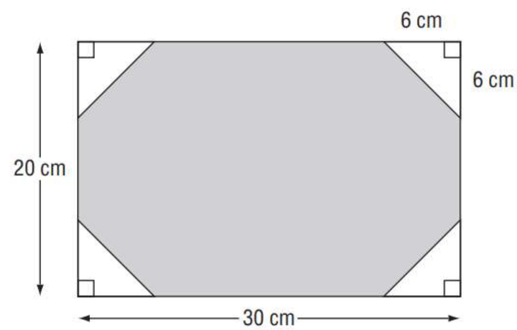
F $x + 14$

G $x + 2$

H $x - 14$

J $x - 2$

38. Bryssa cut four congruent triangles off the corners of a rectangle to make an octagon as shown below.



What is the area of the octagon?

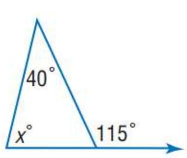
A 456 cm² **C** 552 cm²

B 528 cm² **D** 564 cm²

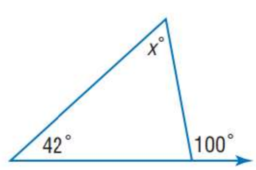
Spiral Review

Find x. (Lesson 4-2)

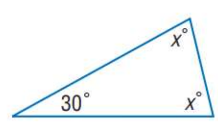
39.



40.



41.



Find x and the measure of each side of the triangle. (Lesson 4-1)

42. $\triangle BCD$ is isosceles with $\overline{BC} \cong \overline{CD}$, $BC = 2x + 4$, $BD = x + 2$ and $CD = 10$.

43. Triangle HKT is equilateral with $HK = x + 7$ and $HT = 4x - 8$.

GET READY for the Next Lesson

PREREQUISITE SKILL **Find the distance between each pair of points.** (Lesson 1-3)

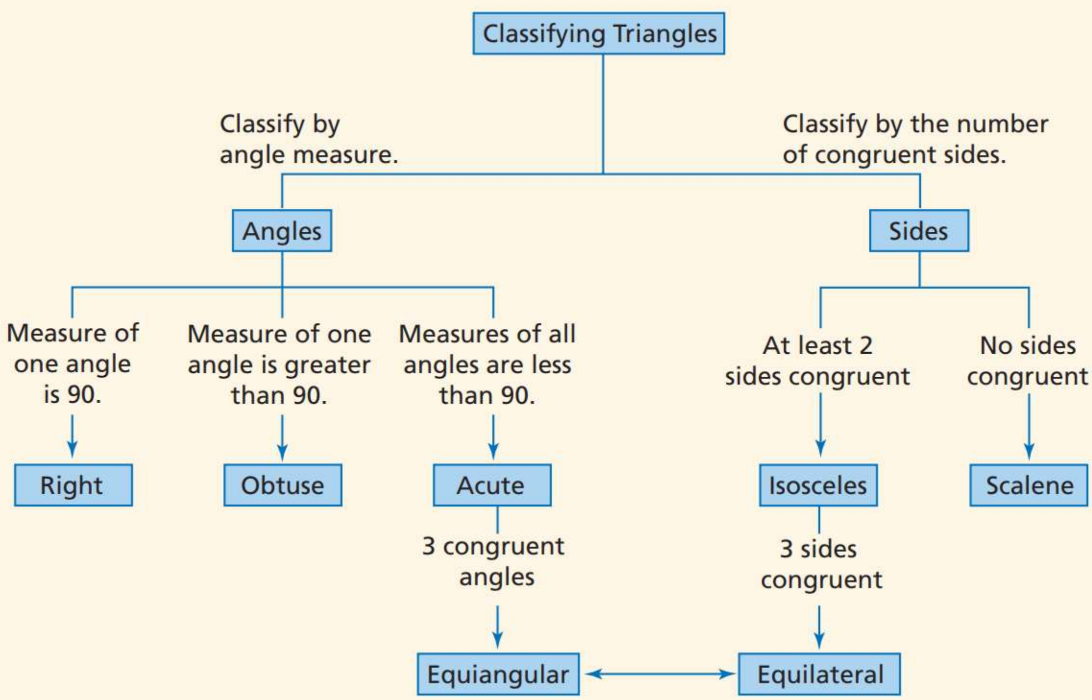
44. $(-1, 7)$, $(1, 6)$ 45. $(8, 2)$, $(4, -2)$ 46. $(3, 5)$, $(5, 2)$ 47. $(0, -6)$, $(-3, -1)$

READING MATH

Making Concept Maps

When studying a chapter, it is wise to record the main topics and vocabulary you encounter. In this chapter, some of the new vocabulary words were *triangle*, *acute triangle*, *obtuse triangle*, *right triangle*, *equiangular triangle*, *scalene triangle*, *isosceles triangle*, and *equilateral triangle*. The triangles are all related by the size of the angles or the number of congruent sides.

A graphic organizer called a *concept map* is a convenient way to show these relationships. A concept map is shown below for the different types of triangles. The main ideas are in boxes. Any information that describes how to move from one box to the next is placed along the arrows.



Reading to Learn

1. Describe how to use the concept map to classify triangles by their side lengths.
2. In $\triangle ABC$, $m\angle A = 48$, $m\angle B = 41$, and $m\angle C = 91$. Use the concept map to classify $\triangle ABC$.
3. Identify the type of triangle that is linked to both classifications.

4-4 Proving Congruence—SSS, SAS

Main Ideas

- Use the SSS Postulate to test for triangle congruence.
- Use the SAS Postulate to test for triangle congruence.

Standard 5.0 Students prove that triangles are congruent or similar, and they are able to use the concept of corresponding parts of congruent triangles.

New Vocabulary

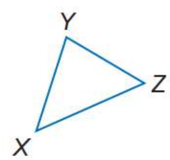
included angle

GET READY for the Lesson

Around 120 B.C., Greek developers and land owners used the properties of geometry to accurately and precisely divide plots of land. Since that time, surveying has been used in areas such as map making and engineering. To check a measurement, land surveyors mark out a right triangle and then mark a second triangle that is congruent to the first.

SSS Postulate Is it always necessary to show that all of the corresponding parts of two triangles are congruent to prove that the triangles are congruent? In this lesson, we will explore two other methods to prove that triangles are congruent.

Use the following construction to construct a triangle with sides that are congruent to a given $\triangle XYZ$.



CONSTRUCTION

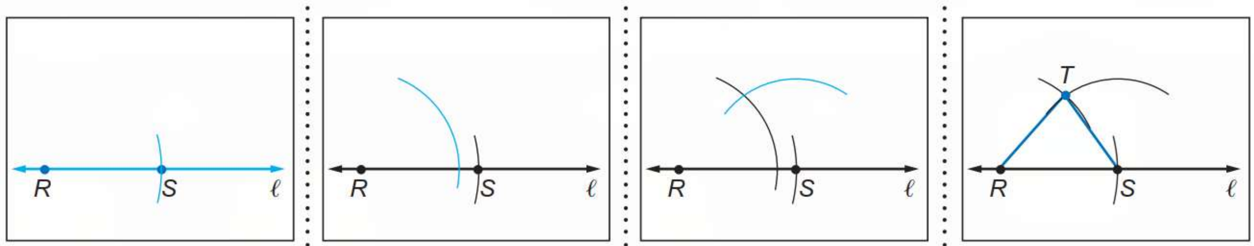
Congruent Triangles Using Sides

Step 1 Use a straightedge to draw any line ℓ, and select a point R. Use a compass to construct $\overline{RS}$ on ℓ, such that $\overline{RS} \cong \overline{XZ}$.

Step 2 Using R as the center, draw an arc with radius equal to XY.

Step 3 Using S as the center, draw an arc with radius equal to YZ.

Step 4 Let T be the point of intersection of the two arcs. Draw $\overline{RT}$ and $\overline{ST}$ to form $\triangle RST$.



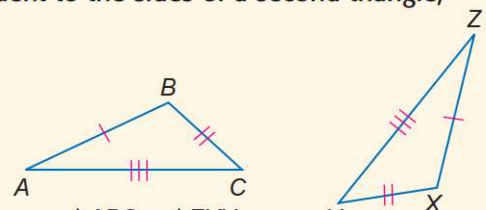
Step 5 Cut out $\triangle RST$ and place it over $\triangle XYZ$. How does $\triangle RST$ compare to $\triangle XYZ$?

If the corresponding sides of two triangles are congruent, then the triangles are congruent. This is the Side-Side-Side Postulate and is written as SSS.

POSTULATE 4.1 **Side-Side-Side Congruence**

If the sides of one triangle are congruent to the sides of a second triangle, then the triangles are congruent.

Abbreviation: *SSS*



$\triangle ABC \cong \triangle ZXY$

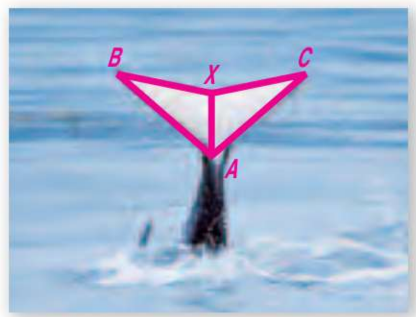
Real-World EXAMPLE Use SSS in Proofs

Real-World Link

Orca whales are commonly called "killer whales" because of their predatory nature. They are the largest members of the dolphin family. An average male is about 19–22 feet long and weighs between 8000 and 12,000 pounds.

Source: seaworld.org

1 **MARINE BIOLOGY** **The tail of an orca whale can be viewed as two triangles that share a common side. Write a two-column proof to prove that $\triangle BXA \cong \triangle CXA$ if $\overline{AB} \cong \overline{AC}$ and $\overline{BX} \cong \overline{CX}$.**



Given: $\overline{AB} \cong \overline{AC}$; $\overline{BX} \cong \overline{CX}$

Prove: $\triangle BXA \cong \triangle CXA$

Proof:

Statements	Reasons
1. $\overline{AB} \cong \overline{AC}$; $\overline{BX} \cong \overline{CX}$	**1.** Given
2. $\overline{AX} \cong \overline{AX}$	**2.** Reflexive Property
3. $\triangle BXA \cong \triangle CXA$	**3.** SSS

CHECK Your Progress

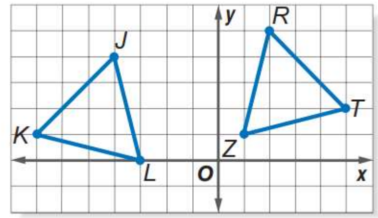
1A. A "Caution, Floor Slippery When Wet" sign is composed of three triangles. If $\overline{AB} \cong \overline{AD}$ and $\overline{CB} \cong \overline{DC}$, prove that $\triangle ACB \cong \triangle ACD$.

1B. Triangle QRS is an isosceles triangle with $\overline{QR} \cong \overline{RS}$. If there exists a line $\overleftrightarrow{RT}$ that bisects $\angle QRS$ and $\overline{QS}$, show that $\triangle QRT \cong \triangle SRT$.

You can use the Distance Formula and postulates about triangle congruence to relate figures on the coordinate plane.

EXAMPLE **SSS on the Coordinate Plane**

2 **COORDINATE GEOMETRY** **Determine whether $\triangle RTZ \cong \triangle JKL$ for $R(2, 5)$, $Z(1, 1)$, $T(5, 2)$, $L(-3, 0)$, $K(-7, 1)$, and $J(-4, 4)$. Explain.**



Use the Distance Formula to show that the corresponding sides are congruent.

$$RT = \sqrt{(2-5)^2 + (5-2)^2} = \sqrt{9+9} = \sqrt{18} \text{ or } 3\sqrt{2}$$

$$JK = \sqrt{[-4-(-7)]^2 + (4-1)^2} = \sqrt{9+9} = \sqrt{18} \text{ or } 3\sqrt{2}$$

$$TZ = \sqrt{(5-1)^2 + (2-1)^2} = \sqrt{16+1} \text{ or } \sqrt{17}$$

$$KL = \sqrt{[-7-(-3)]^2 + (1-0)^2} = \sqrt{16+1} \text{ or } \sqrt{17}$$

$$RZ = \sqrt{(2-1)^2 + (5-1)^2} = \sqrt{1+16} \text{ or } \sqrt{17}$$

$$JL = \sqrt{[-4-(-3)]^2 + (4-0)^2} = \sqrt{1+16} \text{ or } \sqrt{17}$$

$RT = JK$, $TZ = KL$, and $RZ = JL$. By definition of congruent segments, all corresponding segments are congruent. Therefore, $\triangle RTZ \cong \triangle JKL$ by SSS.

CHECK Your Progress

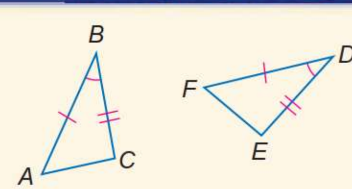
2. Determine whether triangles ABC and TDS with vertices $A(1, 1)$, $B(3, 2)$, $C(2, 5)$, $T(1, -1)$, $D(3, -3)$, and $S(2, -5)$ are congruent. Justify your reasoning.

SAS Postulate Suppose you are given the measures of two sides and the angle they form, called the **included angle**. These conditions describe a unique triangle. Two triangles in which corresponding sides and the included pairs of angles are congruent provide another way to show that triangles are congruent.

POSTULATE 4.2 ***Side-Angle-Side Congruence***

If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent.

Abbreviation: *SAS*



$\triangle ABC \cong \triangle FDE$

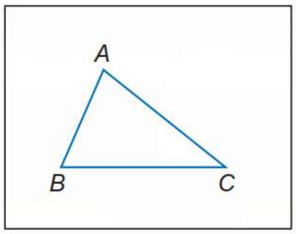
Concepts in Motion
Animation
ca.geometryonline.com

You can also construct congruent triangles given two sides and the included angle.

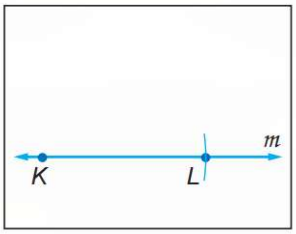
CONSTRUCTION

Congruent Triangles Using Two Sides and the Included Angle

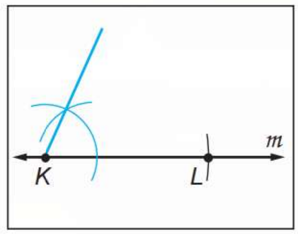
Step 1 Draw a triangle and label its vertices *A*, *B*, and *C*.



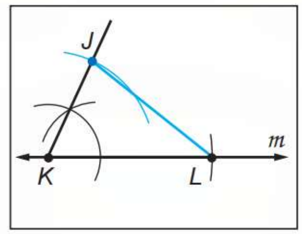
Step 2 Select a point *K* on line *m*. Use a compass to construct $\overline{KL}$ on *m* such that $\overline{KL} \cong \overline{BC}$.



Step 3 Construct an angle congruent to $\angle B$ using $\overrightarrow{KL}$ as a side of the angle and point *K* as the vertex.



Step 4 Construct $\overline{JK}$ such that $\overline{JK} \cong \overline{AB}$. Draw $\overline{JL}$ to complete $\triangle JKL$.



Step 5 Cut out $\triangle JKL$ and place it over $\triangle ABC$. How does $\triangle JKL$ compare to $\triangle ABC$?

Study Tip

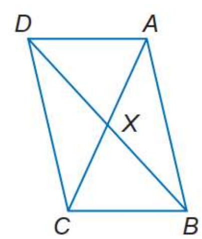
Flow Proofs
Flow proofs can be written vertically or horizontally.

EXAMPLE Use SAS in Proofs

3 **Write a flow proof.**

Given: X is the midpoint of $\overline{BD}$.
X is the midpoint of $\overline{AC}$.

Prove: $\triangle DXC \cong \triangle BXA$



Flow Proof:

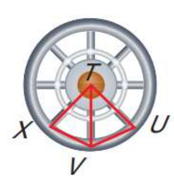
X is the midpoint of $\overline{DB}$. (Given) → $\overline{DX} \cong \overline{BX}$ (Midpoint Theorem) → $\triangle DXC \cong \triangle BXA$ (SAS)

X is the midpoint of $\overline{AC}$. (Given) → $\overline{CX} \cong \overline{AX}$ (Midpoint Theorem) → $\triangle DXC \cong \triangle BXA$ (SAS)

$\angle DXC \cong \angle BXA$ (Vertical ∠s are ≅.) → $\triangle DXC \cong \triangle BXA$ (SAS)

CHECK Your Progress

3. The spokes used in a captain's wheel divide the wheel into eight parts. If $\overline{TU} \cong \overline{TX}$ and $\angle XTV \cong \angle UTV$, show that $\triangle XTV \cong \triangle UTV$.



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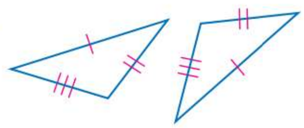
EXAMPLE 4 Identify Congruent Triangles

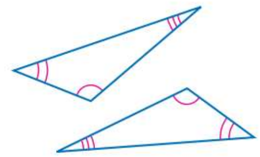
Determine which postulate can be used to prove that the triangles are congruent. If it is not possible to prove that they are congruent, write *not possible*.

a.



Each pair of corresponding sides are congruent. The triangles are congruent by the SSS Postulate.

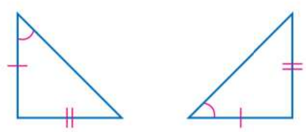
b.



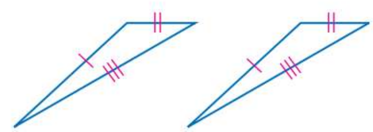
The triangles have three pairs of corresponding angles congruent. This does not match the SSS or the SAS Postulate. It is *not possible* to prove them congruent.

CHECK Your Progress

4A.



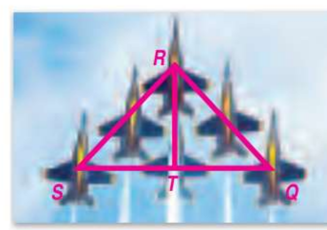
4B.



CHECK Your Understanding

Example 1 (p. 226)

1. JETS The United States Navy Flight Demonstration Squadron, the Blue Angels, fly in a formation that can be viewed as two triangles with a common side. Write a two-column proof to prove that $\triangle SRT \cong \triangle QRT$ if T is the midpoint of $\overline{SQ}$ and $\overline{SR} \cong \overline{QR}$.

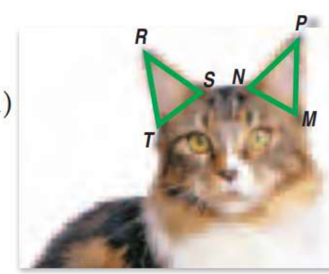


Example 2 (p. 227)

Determine whether $\triangle EFG \cong \triangle MNP$ given the coordinates of the vertices. Explain.

2. $E(-4, -3)$, $F(-2, 1)$, $G(-2, -3)$, $M(4, -3)$, $N(2, 1)$, $P(2, -3)$

3. $E(-2, -2)$, $F(-4, 6)$, $G(-3, 1)$, $M(2, 2)$, $N(4, 6)$, $P(3, 1)$



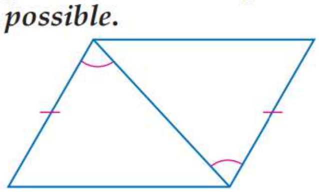
Example 3 (p. 228)

4. CATS A cat's ear is triangular in shape. Write a proof to prove $\triangle RST \cong \triangle PNM$ if $\overline{RS} \cong \overline{PN}$, $\overline{RT} \cong \overline{PM}$, $\angle S \cong \angle N$, and $\angle T \cong \angle M$.

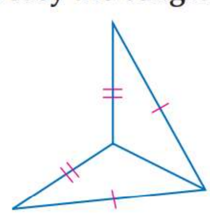
Example 4 (p. 229)

Determine which postulate can be used to prove that the triangles are congruent. If it is not possible to prove that they are congruent, write *not possible*.

5.



6.



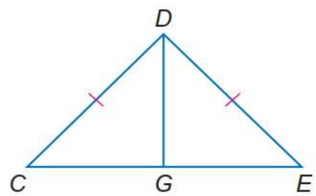
Exercises

HOMEWORK HELP	
For Exercises	See Examples
7, 8	1
9–12	2
13, 14	3
15–18	4

PROOF **For Exercises 7 and 8, write a two-column proof.**

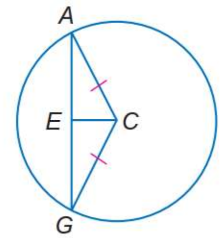
7. Given: $\triangle CDE$ is an isosceles triangle. G is the midpoint of $\overline{CE}$.

Prove: $\triangle CDG \cong \triangle EDG$



8. Given: $\overline{AC} \cong \overline{GC}$
$\overline{EC}$ bisects $\overline{AG}$.

Prove: $\triangle GEC \cong \triangle AEC$



Determine whether $\triangle JKL \cong \triangle FGH$ given the coordinates of the vertices. Explain.

9. $J(2, 5), K(5, 2), L(1, 1), F(-4, 4), G(-7, 1), H(-3, 0)$

10. $J(-1, 1), K(-2, -2), L(-5, -1), F(2, -1), G(3, -2), H(2, 5)$

11. $J(-1, -1), K(0, 6), L(2, 3), F(3, 1), G(5, 3), H(8, 1)$

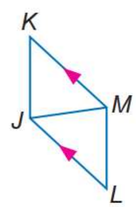
12. $J(3, 9), K(4, 6), L(1, 5), F(1, 7), G(2, 4), H(-1, 3)$

PROOF **For Exercises 13 and 14, write the specified type of proof.**

13. flow proof

Given: $\overline{KM} \parallel \overline{LJ}$, $\overline{KM} \cong \overline{LJ}$

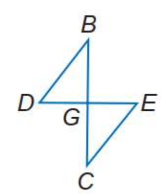
Prove: $\triangle JKM \cong \triangle MLJ$



14. two-column proof

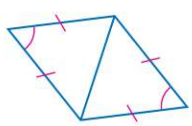
Given: $\overline{DE}$ and $\overline{BC}$ bisect each other.

Prove: $\triangle DGB \cong \triangle EGC$

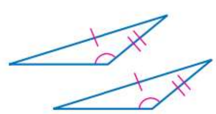


Determine which postulate can be used to prove that the triangles are congruent. If it is not possible to prove that they are congruent, write *not possible*.

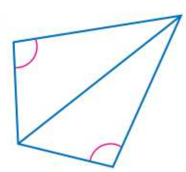
15.



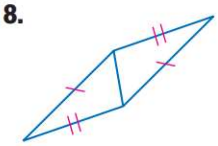
16.



17.



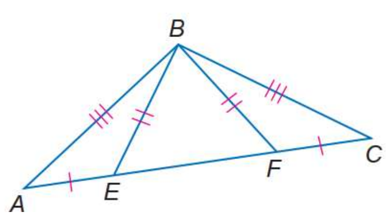
18.



PROOF **For Exercises 19 and 20, write a flow proof.**

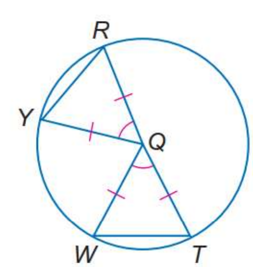
19. Given: $\overline{AE} \cong \overline{CF}$, $\overline{AB} \cong \overline{CB}$, $\overline{BE} \cong \overline{BF}$

Prove: $\triangle AFB \cong \triangle CEB$

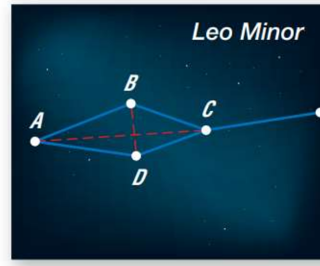


20. Given: $\overline{RQ} \cong \overline{TQ} \cong \overline{YQ} \cong \overline{WQ}$
$\angle RQY \cong \angle WQT$

Prove: $\triangle QWT \cong \triangle QYR$



21. CONSTELLATIONS The "new" constellation of Leo Minor, as envisioned by H.A. Rey in his book *Find the Constellations*, is shown. Write a proof to show that if $\triangle ABD$ is isosceles and $\overline{AC}$ bisects $\angle BAD$, then $BC = CD$.

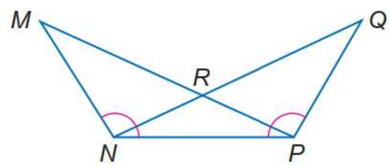


Real-World Link Formerly Edison Field, Angel Stadium in Anaheim, California, opened in 1966 and now has a seating capacity of over 45,000 people. Its infield, like all Major League Baseball fields, is a square 90 feet on each side.

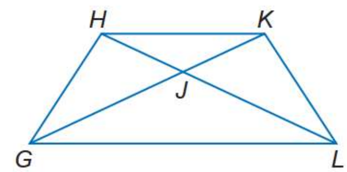
Source: www.ballparks.com

PROOF For Exercises 22 and 23, write a two-column proof.

22. Given: $\triangle MRN \cong \triangle QRP$
$\angle MNP \cong \angle QPN$
Prove: $\triangle MNP \cong \triangle QPN$



23. Given: $\triangle GHJ \cong \triangle LKJ$
Prove: $\triangle GHL \cong \triangle LKG$



BASEBALL For Exercises 24 and 25, use the following information.
A baseball diamond is a square with four right angles and all sides congruent.

24. Write a two-column proof to prove that the distance from first base to third base is the same as the distance from home plate to second base.

25. Write a two-column proof to prove that the angle formed between second base, home plate, and third base is the same as the angle formed between second base, home plate, and first base.

EXTRA PRACTICE
See pages 818, 831.
Math Online
Self-Check Quiz at ca.geometryonline.com

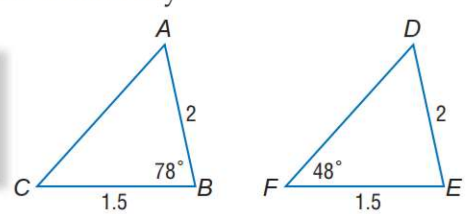
H.O.T. Problems

26. REASONING Explain how the SSS postulate can be used to prove that two triangles are congruent.

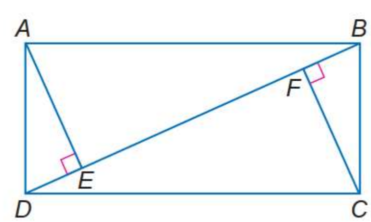
27. OPEN ENDED Find two triangles in a newspaper or magazine and show that they are congruent.

28. FIND THE ERROR Carmelita and Jonathan are trying to determine whether $\triangle ABC$ is congruent to $\triangle DEF$. Who is correct and why?

Carmelita	Jonathan
$\triangle ABC \cong \triangle DEF$ by SAS	Congruence cannot be determined.



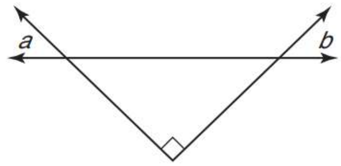
29. CHALLENGE Devise a plan and write a two-column proof for the following.
Given: $\overline{DE} \cong \overline{FB}$, $\overline{AE} \cong \overline{FC}$,
$\overline{AE} \perp \overline{DB}$, $\overline{CF} \perp \overline{DB}$
Prove: $\triangle ABD \cong \triangle CDB$



30. *Writing in Math* Describe two different methods that could be used to prove that two triangles are congruent.

STANDARDS PRACTICE

31. Which of the following statements about the figure is true?



A $a + b < 90$ **C** $a + b = 90$

B $a + b > 90$ **D** $a + b = 45$

32. REVIEW The Murphy family just drove 300 miles to visit their grandparents. Mr. Murphy drove 70 mph for 65% of the trip and 35 mph or less for 20% of the trip that was left. Assuming that Mr. Murphy never went over 70 mph, how many miles did he travel at a speed between 35 and 70 mph?

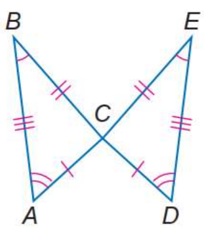
F 195 **H** 21

G 84 **J** 18

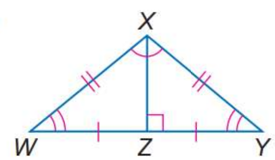
Spiral Review

Identify the congruent triangles in each figure. (Lesson 4-3)

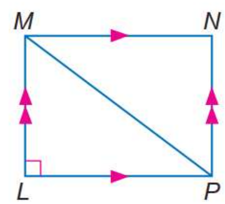
33.



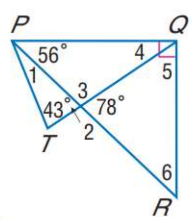
34.



35.



Find each measure if $\overline{PQ} \perp \overline{QR}$. (Lesson 4-2)

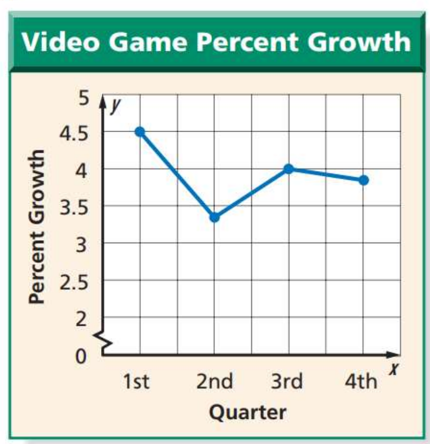


36. $m\angle 2$ **37.** $m\angle 3$

38. $m\angle 5$ **39.** $m\angle 4$

40. $m\angle 1$ **41.** $m\angle 6$

ANALYZE GRAPHS **For Exercises 42 and 43, use the graph of sales of a certain video game system in a recent year.** (Lesson 3-3)

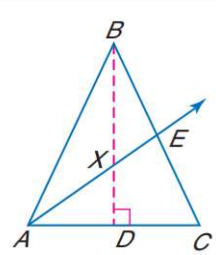


42. Find the rate of change from first quarter to the second quarter.

43. Which had the greater rate of change: first quarter to second quarter, or third to fourth?

GET READY for the Next Lesson

PREREQUISITE SKILL **$\overline{BD}$ and $\overline{AE}$ are angle bisectors and segment bisectors. Name the indicated segments and angles.** (Lessons 1-5 and 1-6)



44. segment congruent to $\overline{EC}$ **45.** angle congruent to $\angle ABD$

46. angle congruent to $\angle BDC$ **47.** segment congruent to $\overline{AD}$

48. angle congruent to $\angle BAE$ **49.** angle congruent to $\angle BXA$

4 Mid-Chapter Quiz
Lessons 4-1 through 4-4

1. **MULTIPLE CHOICE** Classify $\triangle ABC$ with vertices $A(-1, 1)$, $B(1, 3)$, and $C(3, -1)$. (Lesson 4-1)

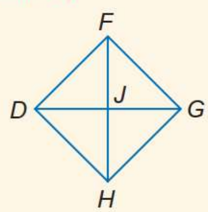
A scalene acute

B equilateral

C isosceles acute

D isosceles right

2. Identify the isosceles triangles in the figure, if $\overline{FH}$ and $\overline{DG}$ are congruent perpendicular bisectors. (Lesson 4-1)



$\triangle ABC$ is equilateral with $AB = 2x$, $BC = 4x - 7$, and $AC = x + 3.5$. (Lesson 4-1)

3. Find x.

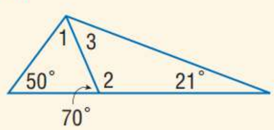
4. Find the measure of each side.

Find the measure of each angle listed below. (Lesson 4-2)

5. $m\angle 1$

6. $m\angle 2$

7. $m\angle 3$

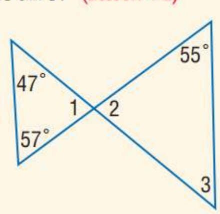


Find each measure. (Lesson 4-2)

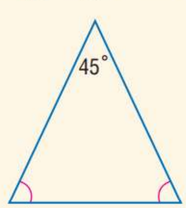
8. $m\angle 1$

9. $m\angle 2$

10. $m\angle 3$



11. Find the missing angle measures. (Lesson 4-2)



12. If $\triangle MNP \cong \triangle JKL$, name the corresponding congruent angles and sides. (Lesson 4-3)

13. **MULTIPLE CHOICE** Given: $\triangle ABC \cong \triangle XYZ$. Which of the following *must* be true? (Lesson 4-3)

F $\angle A \cong \angle Y$

G $\overline{AC} \cong \overline{XZ}$

H $\overline{AB} \cong \overline{YZ}$

J $\angle Z \cong \angle B$

COORDINATE GEOMETRY **The vertices of $\triangle JKL$ are $J(7, 7)$, $K(3, 7)$, $L(7, 1)$. The vertices of $\triangle J'K'L'$ are $J'(7, -7)$, $K'(3, -7)$, $L'(7, -1)$.** (Lesson 4-3)

14. Verify that $\triangle JKL \cong \triangle J'K'L'$.

15. Name the congruence transformation for $\triangle JKL$ and $\triangle J'K'L'$.

16. Determine whether $\triangle JML \cong \triangle BDG$ given that $J(-4, 5)$, $M(-2, 6)$, $L(-1, 1)$, $B(-3, -4)$, $D(-4, -2)$, and $G(1, -1)$. (Lesson 4-4)

Determine whether $\triangle XYZ \cong \triangle TUV$ given the coordinates of the vertices. Explain. (Lesson 4-4)

17. $X(0, 0)$, $Y(3, 3)$, $Z(0, 3)$, $T(-6, -6)$, $U(-3, -3)$, $V(-3, -6)$

18. $X(7, 0)$, $Y(5, 4)$, $Z(1, 1)$, $T(-5, -4)$, $U(-3, 4)$, $V(1,1)$

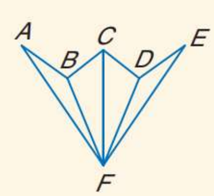
19. $X(9, 6)$, $Y(3, 7)$, $Z(9, -6)$, $T(-10, 7)$, $U(-4, 7)$, $V(-10, -7)$

Write a two-column proof. (Lesson 4-4)

20. **Given:** $\triangle ABF \cong \triangle EDF$

$\overline{CF}$ is angle bisector of $\angle DFB$.

Prove: $\triangle BCF \cong \triangle DCF$.



4-5 Proving Congruence—ASA, AAS

Main Ideas

- Use the ASA Postulate to test for triangle congruence.
- Use the AAS Theorem to test for triangle congruence.

Standard 5.0 Students prove that triangles are congruent or similar, **and they are able to use the concept of corresponding parts of congruent triangles.**

New Vocabulary

included side

GET READY for the Lesson

The Bank of China Tower in Hong Kong has triangular trusses for structural support. These trusses form congruent triangles. In this lesson, we will explore two additional methods of proving triangles congruent.

ASA Postulate Suppose you were given the measures of two angles of a triangle and the side between them, the **included side**. Do these measures form a unique triangle?

CONSTRUCTION

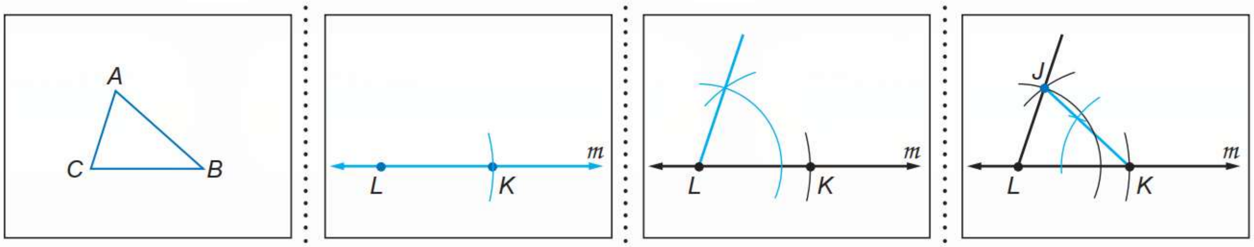
Congruent Triangles Using Two Angles and Included Side

Step 1 Draw a triangle and label its vertices *A*, *B*, and *C*.

Step 2 Draw any line *m* and select a point *L*. Construct $\overline{LK}$ such that $\overline{LK} \cong \overline{CB}$.

Step 3 Construct an angle congruent to $\angle C$ at *L* using $\overrightarrow{LK}$ as a side of the angle.

Step 4 Construct an angle congruent to $\angle B$ at *K* using $\overrightarrow{LK}$ as a side of the angle. Label the point where the new sides of the angles meet *J*.



Step 5 Cut out $\triangle JKL$ and place it over $\triangle ABC$. How does $\triangle JKL$ compare to $\triangle ABC$?

This construction leads to the Angle-Side-Angle Postulate, written as ASA.

Sylvain Grandadam/Photo Researchers

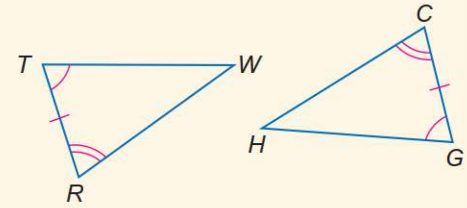
Reading Math

Included Side The *included side* refers to the side that each of the angles share.

POSTULATE 4.3 **Angle-Side-Angle Congruence**

If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.

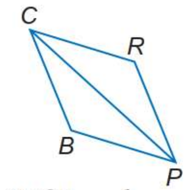
Abbreviation: *ASA*



$\triangle RTW \cong \triangle CGH$

EXAMPLE Use ASA in Proofs

1 **Write a paragraph proof.**

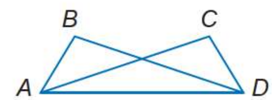


Given: $\overline{CP}$ bisects $\angle BCR$ and $\angle BPR$.

Prove: $\triangle BCP \cong \triangle RCP$

Proof: Since $\overline{CP}$ bisects $\angle BCR$ and $\angle BPR$, $\angle BCP \cong \angle RCP$ and $\angle BPC \cong \angle RPC$. $\overline{CP} \cong \overline{CP}$ by the Reflexive Property. By ASA, $\triangle BCP \cong \triangle RCP$.

CHECK Your Progress



1. Given: $\angle CAD \cong \angle BDA$ and $\angle CDA \cong \angle BAD$

Prove: $\triangle ABD \cong \triangle DCA$

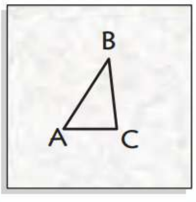
AAS Theorem Suppose you are given the measures of two angles and a nonincluded side. Is this information sufficient to prove two triangles congruent?

GEOMETRY LAB

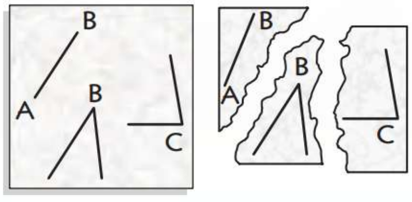
Angle-Angle-Side Congruence

MODEL

Step 1 Draw a triangle on a piece of patty paper. Label the vertices *A*, *B*, and *C*.



Step 2 Copy $\overline{AB}$, $\angle B$, and $\angle C$ on another piece of patty paper and cut them out.



Step 3 Assemble them to form a triangle in which the side is not the included side of the angles.

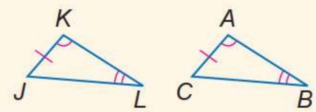
ANALYZE

1. Place the original $\triangle ABC$ over the assembled figure. How do the two triangles compare?
2. **Make a conjecture** about two triangles with two angles and the nonincluded side of one triangle congruent to two angles and the nonincluded side of the other triangle.

This lab leads to the Angle-Angle-Side Theorem, written as AAS.

THEOREM 4.5 — Angle-Angle-Side Congruence

If two angles and a nonincluded side of one triangle are congruent to the corresponding two angles and side of a second triangle, then the two triangles are congruent.



Abbreviation: *AAS*

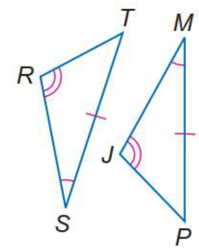
Example: $\triangle JKL \cong \triangle CAB$

PROOF Theorem 4.5

Given: $\angle M \cong \angle S$, $\angle J \cong \angle R$, $\overline{MP} \cong \overline{ST}$

Prove: $\triangle JMP \cong \triangle RST$

Proof:



Statements	Reasons
1. $\angle M \cong \angle S$, $\angle J \cong \angle R$, $\overline{MP} \cong \overline{ST}$	**1.** Given
2. $\angle P \cong \angle T$	**2.** Third Angle Theorem
3. $\triangle JMP \cong \triangle RST$	**3.** ASA

EXAMPLE Use AAS in Proofs

Study Tip

Overlapping Triangles

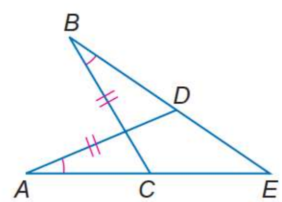
When triangles overlap, it is a good idea to draw each triangle separately and label the congruent parts.

2 Write a flow proof.

Given: $\angle EAD \cong \angle EBC$

$\overline{AD} \cong \overline{BC}$

Prove: $\overline{AE} \cong \overline{BE}$



Flow Proof:

$\angle EAD \cong \angle EBC$ — **Given**

$\overline{AD} \cong \overline{BC}$ — **Given**

$\angle E \cong \angle E$ — Reflexive Property

→ $\triangle ADE \cong \triangle BCE$ — AAS

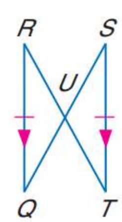
→ $\overline{AE} \cong \overline{BE}$ — CPCTC

CHECK Your Progress

2. Write a flow proof.

Given: $\overline{RQ} \cong \overline{ST}$ and $\overline{RQ} \parallel \overline{ST}$

Prove: $\triangle RUQ \cong \triangle TUS$



You have learned several methods for proving triangle congruence. The Concept Summary lists ways to help you determine which method to use.

CONCEPT SUMMARY

Method	Use when. . .
Definition of Congruent Triangles	All corresponding parts of one triangle are congruent to the corresponding parts of the other triangle.
SSS	The three sides of one triangle are congruent to the three sides of the other triangle.
SAS	Two sides and the included angle of one triangle are congruent to two sides and the included angle of the other triangle.
ASA	Two angles and the included side of one triangle are congruent to two angles and the included side of the other triangle.
AAS	Two angles and a nonincluded side of one triangle are congruent to two angles and side of the other triangle.

Real-World Career

Architect

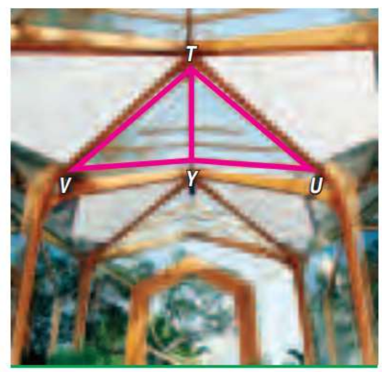
About 28% of architects are self-employed. Architects design a variety of buildings including offices, retail spaces, and schools.

Math Online

For more information, go to ca.geometryonline.com.

Real-World EXAMPLE — Determine if Triangles Are Congruent

3 **ARCHITECTURE This glass chapel was designed by Frank Lloyd Wright's son, Lloyd Wright. Suppose the redwood supports, $\overline{TU}$ and $\overline{TV}$, measure 3 feet, $TY = 1.6$ feet, and $m\angle U$ and $m\angle V$ are 31. Determine whether $\triangle TYU \cong \triangle TYV$. Justify your answer.**

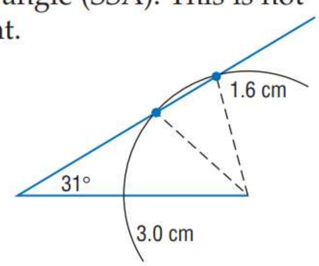


Explore We are given three measurements of each triangle. We need to determine whether the two triangles are congruent.

Plan Since $m\angle U = m\angle V$, $\angle U \cong \angle V$. Likewise, $TU = TV$ so $\overline{TU} \cong \overline{TV}$, and $TY = TY$ so $\overline{TY} \cong \overline{TY}$. Check each possibility using the five methods you know.

Solve We are given information about side-side-angle (SSA). This is not a method to prove two triangles congruent.

Check Use a compass, protractor, and ruler to draw a triangle with the given measurements. For space purposes, use centimeters instead of feet.



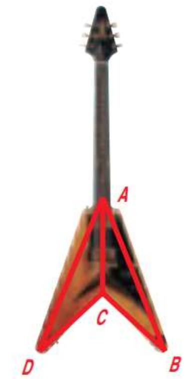
- Draw a segment 3.0 centimeters long.
- At one end, draw an angle of 31°. Extend the line longer than 3.0 centimeters.
- At the other end, draw an arc with a radius of 1.6 centimeters such that it intersects the line.

Notice that there are two possible segments that could determine the triangle. Since the given measurements do not lead to a unique triangle, we cannot show that the triangles are congruent.

(continued on the next page)

CHECK Your Progress

3. A flying V guitar is made up of two triangles. If $AB = 27$ inches, $AD = 27$ inches, $DC = 7$ inches, and $CB = 7$ inches, determine whether $\triangle ADC \cong \triangle ABC$. Explain.



Concepts in Motion
Interactive Lab
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Online **Personal Tutor at** ca.geometryonline.com

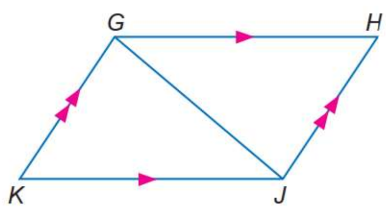
CHECK Your Understanding

Example 1 (p. 235)

PROOF **For Exercises 1–4, write the specified type of proof.**

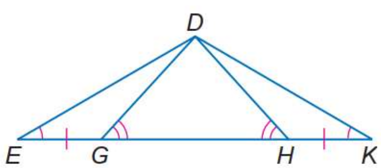
1. flow proof

Given: $\overline{GH} \parallel \overline{KJ}$, $\overline{GK} \parallel \overline{HJ}$
Prove: $\triangle GJK \cong \triangle JGH$



2. paragraph proof

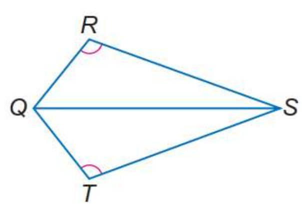
Given: $\angle E \cong \angle K$, $\angle DGH \cong \angle DHG$
$\overline{EG} \cong \overline{KH}$
Prove: $\triangle EGD \cong \triangle KHD$



Example 2 (p. 236)

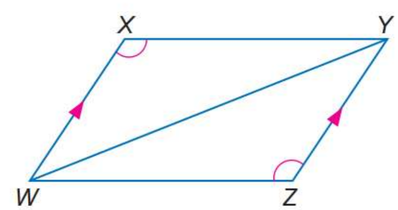
3. paragraph proof

Given: $\overline{QS}$ bisects $\angle RST$; $\angle R \cong \angle T$
Prove: $\triangle QRS \cong \triangle QTS$



4. flow proof

Given: $\overline{XW} \parallel \overline{YZ}$, $\angle X \cong \angle Z$
Prove: $\triangle WXY \cong \triangle YZW$



Example 3 (p. 237)

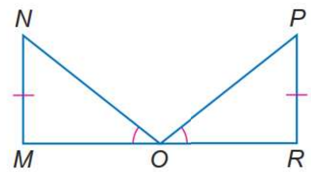
5. PARACHUTES Suppose $\overline{ST}$ and $\overline{ML}$ each measure seven feet, $\overline{SR}$ and $\overline{MK}$ each measure 5.5 feet, and $m\angle T = m\angle L = 49$. Determine whether $\triangle SRT \cong \triangle MKL$. Justify your answer.

Exercises

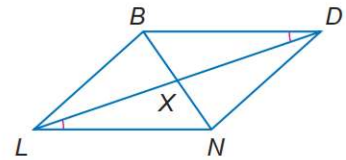
HOMEWORK HELP	
For Exercises	See Examples
6, 7	1
8, 9	2
10, 11	3

Write a paragraph proof.

6. Given: $\angle NOM \cong \angle POR$, $\overline{NM} \perp \overline{MR}$, $\overline{PR} \perp \overline{MR}$, $\overline{NM} \cong \overline{PR}$
Prove: $\overline{MO} \cong \overline{OR}$

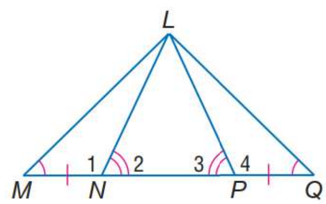


7. Given: $\overline{DL}$ bisects $\overline{BN}$. $\angle XLN \cong \angle XDB$
Prove: $\overline{LN} \cong \overline{DB}$

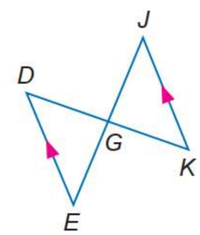


Write a flow proof.

8. Given: $\overline{MN} \cong \overline{PQ}$, $\angle M \cong \angle Q$, $\angle 2 \cong \angle 3$
Prove: $\triangle MLP \cong \triangle QLN$



9. Given: $\overline{DE} \parallel \overline{JK}$, $\overline{DK}$ bisects $\overline{JE}$.
Prove: $\triangle EGD \cong \triangle JGK$



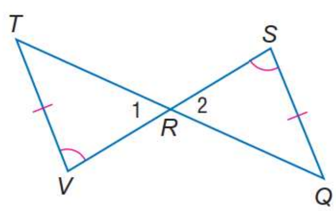
GARDENING For Exercises 10 and 11, use the following information.
Beth is planning a garden. She wants the triangular sections $\triangle CFD$ and $\triangle HFG$ to be congruent. F is the midpoint of $\overline{DG}$, and $DG = 16$ feet.

10. Suppose $\overline{CD}$ and $\overline{GH}$ each measure 4 feet and the measure of $\angle CFD$ is 29. Determine whether $\triangle CFD \cong \triangle HFG$. Justify your answer.

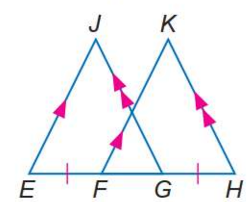
11. Suppose F is the midpoint of $\overline{CH}$, and $\overline{CH} \cong \overline{DG}$. Determine whether $\triangle CFD \cong \triangle HFG$. Justify your answer.

Write a flow proof.

12. Given: $\angle V \cong \angle S$, $\overline{TV} \cong \overline{QS}$
Prove: $\overline{VR} \cong \overline{SR}$

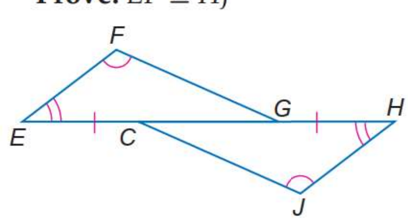


13. Given: $\overline{EJ} \parallel \overline{FK}$, $\overline{JG} \parallel \overline{KH}$, $\overline{EF} \cong \overline{GH}$
Prove: $\triangle EJG \cong \triangle FKH$

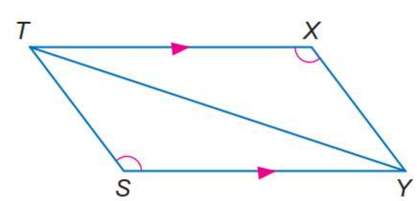


Write a paragraph proof.

14. Given: $\angle F \cong \angle J$, $\angle E \cong \angle H$, $\overline{EC} \cong \overline{GH}$
Prove: $\overline{EF} \cong \overline{HJ}$



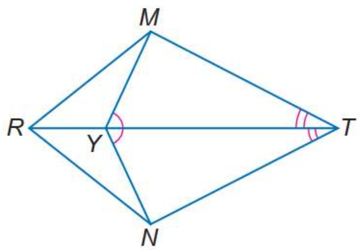
15. Given: $\overline{TX} \parallel \overline{SY}$, $\angle TXY \cong \angle TSY$
Prove: $\triangle TSY \cong \triangle YXT$



PROOF **Write a two-column proof.**

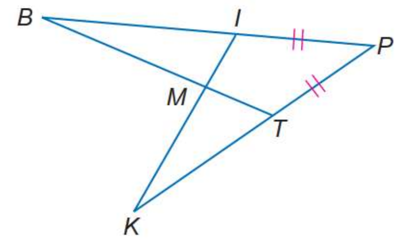
16. **Given:** $\angle MYT \cong \angle NYT$, $\angle MTY \cong \angle NTY$

Prove: $\triangle RYM \cong \triangle RYN$



17. **Given:** $\triangle BMI \cong \triangle KMT$, $\overline{IP} \cong \overline{PT}$

Prove: $\triangle IPK \cong \triangle TPB$

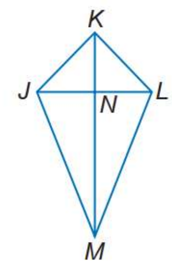


Real-World Link

The largest kite ever flown was 210 feet long and 72 feet wide.

Source: *Guinness Book of World Records*

KITES **For Exercises 18 and 19, use the following information.** Austin is making a kite. Suppose *JL* is two feet, *JM* is 2.7 feet, and the measure of $\angle NJM$ is 68.



18. If *N* is the midpoint of $\overline{JL}$ and $\overline{KM} \perp \overline{JL}$, determine whether $\triangle JKN \cong \triangle LKN$. Justify your answer.

19. If $\overline{JM} \cong \overline{LM}$ and $\angle NJM \cong \angle NLM$, determine whether $\triangle JNM \cong \triangle LNM$. Justify your answer.

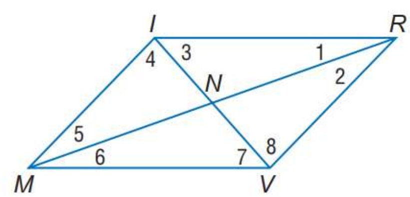
EXTRA PRACTICE

See pages 808, 831.

Math Online

Self-Check Quiz at ca.geometryonline.com

Complete each congruence statement and the postulate or theorem that applies.



20. If $\overline{IM} \cong \overline{RV}$ and $\angle 2 \cong \angle 5$, then $\triangle INM \cong \triangle$ __?__ by __?__.

21. If $\overline{IR} \parallel \overline{MV}$ and $\overline{IR} \cong \overline{MV}$, then $\triangle IRN \cong \triangle$ __?__ by __?__.

H.O.T. Problems

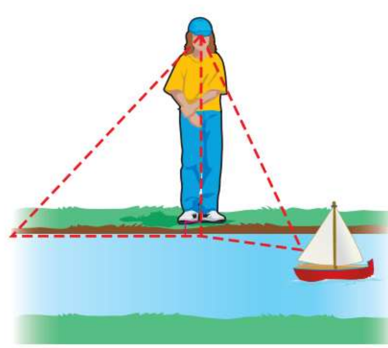
22. **Which One Doesn't Belong?** Identify the term that does not belong with the others. Explain your reasoning.

ASA	SSS	SSA	AAS

23. **REASONING** Find a counterexample to show why AAA (Angle-Angle-Angle) cannot be used to prove congruence in triangles.

24. **OPEN ENDED** Draw and label two triangles that could be proved congruent by SAS.

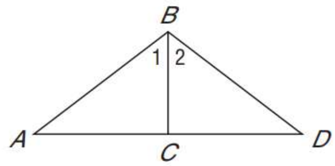
25. **CHALLENGE** Neva wants to estimate the distance between herself and a toy boat. She adjusts the visor of her cap so that it is in line with her line of sight to the toy boat. She keeps her neck stiff and turns her body to establish a line of sight to a point on the ground. Then she paces out the distance to the new point. Is the distance from the toy boat the same as the distance she just paced out? Explain your reasoning.



26. ***Writing in Math*** Use the information about construction on page 234 to explain how congruent triangles are used in construction. Include why it is important to use congruent triangles for support.

STANDARDS PRACTICE

27. **Given:** $\overline{BC}$ is perpendicular to $\overline{AD}$; $\angle 1 \cong \angle 2$.



Which theorem or postulate could be used to prove $\triangle ABC \cong \triangle DBC$?

A AAS

B ASA

C SAS

D SSS

28. **REVIEW** Which expression can be used to find the values of $s(n)$ in the table?

n	−8	−4	−1	0	1
s(n)	1.00	2.00	2.75	3.00	3.25

F $-2n + 3$

G $-n + 7$

H $\frac{1}{4}n + 3$

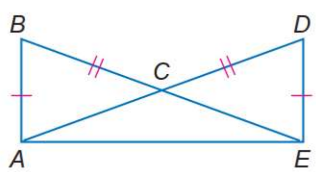
J $\frac{1}{2}n + 5$

Spiral Review

Write a flow proof. (Lesson 4-4)

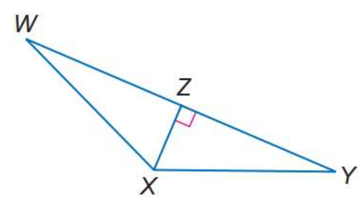
29. **Given:** $\overline{BA} \cong \overline{DE}$, $\overline{DA} \cong \overline{BE}$

Prove: $\triangle BEA \cong \triangle DAE$



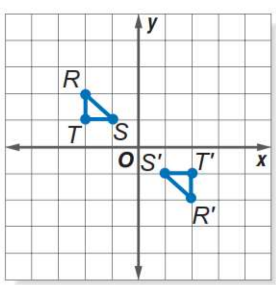
30. **Given:** $\overline{XZ} \perp \overline{WY}$, $\overline{XZ}$ bisects $\overline{WY}$.

Prove: $\triangle WZX \cong \triangle YZX$

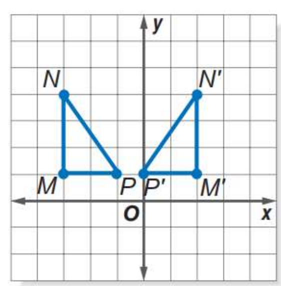


Verify congruence and name the congruence transformation. (Lesson 4-3)

31. $\triangle RTS \cong \triangle R'T'S'$



32. $\triangle MNP \cong \triangle M'N'P'$



Write each statement in if-then form. (Lesson 2-3)

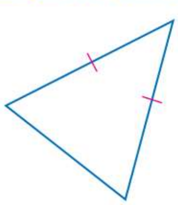
33. Happy people rarely correct their faults.

34. A champion is afraid of losing.

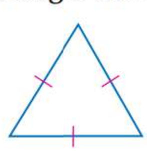
GET READY for the Next Lesson

PREREQUISITE SKILL **Classify each triangle according to its sides.** (Lesson 4-1)

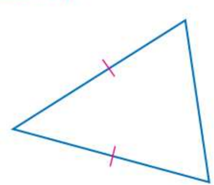
35.



36.



37.



EXTEND 4-5

Geometry Lab

Congruence in Right Triangles

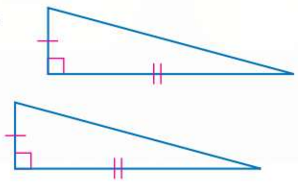
Standard 5.0 Students prove that triangles are congruent or similar, **and they are able to use the concept of corresponding parts of congruent triangles.**

In Lessons 4-4 and 4-5, you learned theorems and postulates to prove triangles congruent. Do these theorems and postulates apply to right triangles?

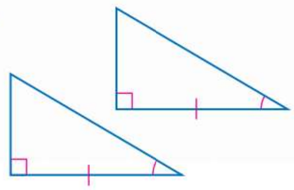
ACTIVITY 1 Triangle Congruence

Study each pair of right triangles.

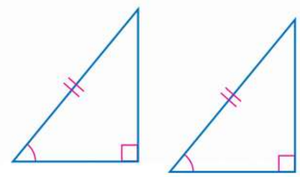
a.



b.



c.



ANALYZE THE RESULTS

1. Is each pair of triangles congruent? If so, which congruence theorem or postulate applies?
2. Rewrite the congruence rules from Exercise 1 using *leg*, (L), or *hypotenuse*, (H), to replace *side*. Omit the A for any right angle since we know that all right triangles contain a right angle and all right angles are congruent.
3. **MAKE A CONJECTURE** If you know that the corresponding legs of two right triangles are congruent, what other information do you need to declare the triangles congruent? Explain.

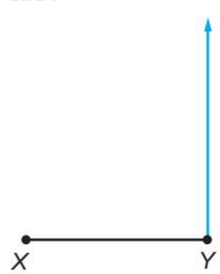
In Lesson 4-5, you learned that SSA is not a valid test for determining triangle congruence. Can SSA be used to prove right triangles congruent?

ACTIVITY 2 SSA and Right Triangles

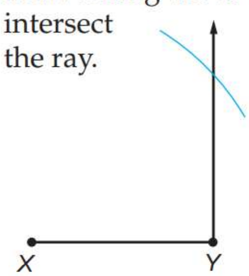
How many right triangles exist that have a hypotenuse of 10 centimeters and a leg of 7 centimeters?

Step 1
Draw $\overline{XY}$ so that $XY = 7$ centimeters.

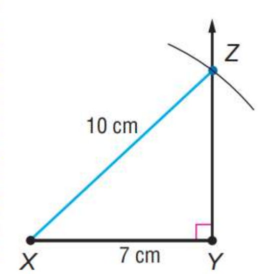
Step 2
Use a protractor to draw a ray from Y that is perpendicular to $\overline{XY}$.



Step 3
Open your compass to a width of 10 centimeters. Place the point at X and draw a long arc to intersect the ray.



Step 4
Label the intersection Z and draw $\overline{XZ}$ to complete $\triangle XYZ$.

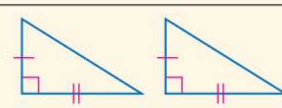
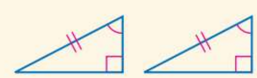
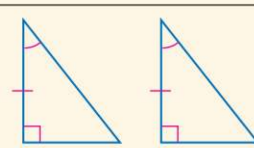
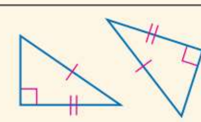


ANALYZE THE RESULTS

4. Does the model yield a unique triangle?
5. Can you use the lengths of the hypotenuse and a leg to show right triangles are congruent?
6. **Make a conjecture** about the case of SSA that exists for right triangles.

The two activities provide evidence for four ways to prove right triangles congruent.

KEY CONCEPT — *Right Triangle Congruence*

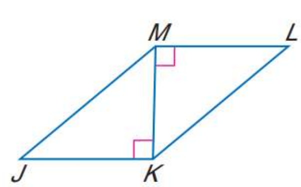
Theorems	Abbreviation	Example
4.6 Leg-Leg Congruence If the legs of one right triangle are congruent to the corresponding legs of another right triangle, then the triangles are congruent.	**LL**	
4.7 Hypotenuse-Angle Congruence If the hypotenuse and acute angle of one right triangle are congruent to the hypotenuse and corresponding acute angle of another right triangle, then the two triangles are congruent.	**HA**	
4.8 Leg-Angle Congruence If one leg and an acute angle of one right triangle are congruent to the corresponding leg and acute angle of another right triangle, then the triangles are congruent.	**LA**	
Postulate		
4.4 Hypotenuse-Leg Congruence If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and corresponding leg of another right triangle, then the triangles are congruent.	**HL**	

EXERCISES

PROOF Write a paragraph proof of each theorem.

7. Theorem 4.6
8. Theorem 4.7
9. Theorem 4.8 (*Hint*: There are two possible cases.)

Use the figure to write a two-column proof.



10. **Given:** $\overline{ML} \perp \overline{MK}$, $\overline{JK} \perp \overline{KM}$
$\angle J \cong \angle L$
Prove: $\overline{JM} \cong \overline{KL}$

11. **Given:** $\overline{JK} \perp \overline{KM}$, $\overline{JM} \cong \overline{KL}$
$\overline{ML} \parallel \overline{JK}$
Prove: $\overline{ML} \cong \overline{JK}$

4-6 Isosceles Triangles

Main Ideas

- Use properties of isosceles triangles.
- Use properties of equilateral triangles.

Standard 4.0 **Students prove basic theorems involving congruence** and similarity. (Key)

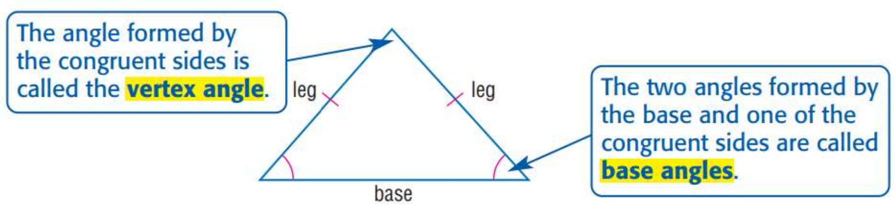
New Vocabulary

vertex angle
base angles

GET READY for the Lesson

The art of Lois Mailou Jones, a twentieth-century artist, includes paintings and textile design, as well as book illustration. Notice the isosceles triangles in this painting, *Damballah*.

Properties of Isosceles Triangles In Lesson 4-1, you learned that isosceles triangles have two congruent sides. Like the right triangle, the parts of an isosceles triangle have special names.

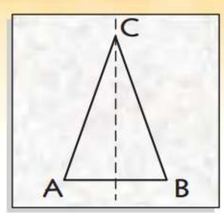


GEOMETRY LAB

Isosceles Triangles

MODEL

- Draw an acute triangle on patty paper with $\overline{AC} \cong \overline{BC}$.
- Fold the triangle through C so that A and B coincide.



ANALYZE

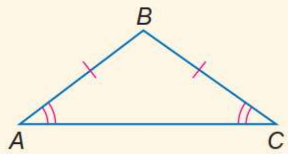
1. What do you observe about $\angle A$ and $\angle B$?
2. Draw an obtuse isosceles triangle. Compare the base angles.
3. Draw a right isosceles triangle. Compare the base angles.

The results of the Geometry Lab suggest Theorem 4.9.

THEOREM 4.9 — Isosceles Triangle

If two sides of a triangle are congruent, then the angles opposite those sides are congruent.

Example: If $\overline{AB} \cong \overline{CB}$, then $\angle A \cong \angle C$.

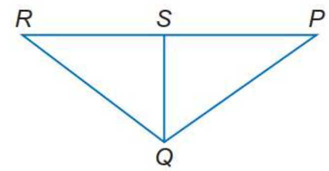


EXAMPLE — Proof of Theorem

1 **Write a two-column proof of the Isosceles Triangle Theorem.**

Given: $\angle PQR$, $\overline{PQ} \cong \overline{RQ}$

Prove: $\angle P \cong \angle R$



Proof:

Statements	Reasons
1. Let S be the midpoint of $\overline{PR}$.	**1.** Every segment has exactly one midpoint.
2. Draw an auxiliary segment $\overline{QS}$	**2.** Two points determine a line.
3. $\overline{PS} \cong \overline{RS}$	**3.** Midpoint Theorem
4. $\overline{QS} \cong \overline{QS}$	**4.** Congruence of segments is reflexive.
5. $\overline{PQ} \cong \overline{RQ}$	**5.** Given
6. $\triangle PQS \cong \triangle RQS$	**6.** SSS
7. $\angle P \cong \angle R$	**7.** CPCTC

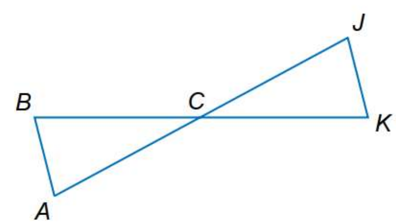
CHECK Your Progress

1. Write a two-column proof.

Given: $\overline{CA} \cong \overline{BC}$; $\overline{KC} \cong \overline{CJ}$

C is the midpoint of $\overline{BK}$.

Prove: $\triangle ABC \cong \triangle JKC$



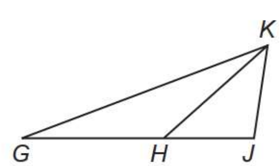
Test-Taking Tip

Diagrams Label the diagram with the given information. Use your drawing to plan the next step in solving the problem.

STANDARDS EXAMPLE — Find a Missing Angle Measure

2 **If $\overline{GH} \cong \overline{HK}$, $\overline{HJ} \cong \overline{JK}$, and $m\angle GJK = 100$, what is $m\angle HGK$?**

A 10 **B** 15 **C** 20 **D** 25



Read the Item

$\triangle GHK$ is isosceles with base $\overline{GK}$. Likewise, $\triangle HJK$ is isosceles with base $\overline{HK}$.

(continued on the next page)

Solve the Item

Step 1 The base angles of $\triangle HJK$ are congruent. Let $x = m\angle KHJ = m\angle HKJ$.

$m\angle KHJ + m\angle HKJ + m\angle HJK = 180$ Angle Sum Theorem

$x + x + 100 = 180$ Substitution

$2x + 100 = 180$ Add.

$2x = 80$ Subtract 100 from each side.

$x = 40$ So, $m\angle KHJ = m\angle HKJ = 40$.

Step 2 $\angle GHK$ and $\angle KHJ$ form a linear pair. Solve for $m\angle GHK$.

$m\angle KHJ + m\angle GHK = 180$ Linear pairs are supplementary.

$40 + m\angle GHK = 180$ Substitution

$m\angle GHK = 140$ Subtract 40 from each side.

Step 3 The base angles of $\triangle GHK$ are congruent. Let y represent $m\angle HGK$ and $m\angle GKH$.

$m\angle GHK + m\angle HGK + m\angle GKH = 180$ Angle Sum Theorem

$140 + y + y = 180$ Substitution

$140 + 2y = 180$ Add.

$2y = 40$ Subtract 140 from each side.

$y = 20$ Divide each side by 2.

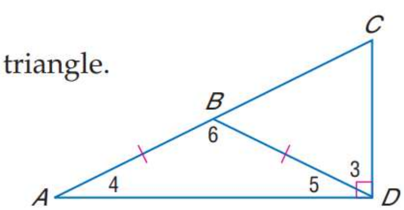
The measure of $\angle HGK$ is 20. Choice C is correct.

CHECK Your Progress

2. $\triangle ABD$ is isosceles, and $\triangle ACD$ is a right triangle. If $m\angle 6 = 136$, what is $m\angle 3$?

F 21 **H** 68

G 37 **J** 113



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Study Tip

Look Back
You can review **converses** in Lesson 2-3.

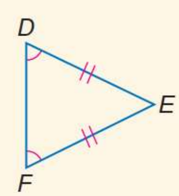
The converse of the Isosceles Triangle Theorem is also true.

THEOREM 4.10

If two angles of a triangle are congruent, then the sides opposite those angles are congruent.

Abbreviation: *Conv. of Isos. △ Th.*

Example: If $\angle D \cong \angle F$, then $\overline{DE} \cong \overline{FE}$.



You will prove Theorem 4.10 in Exercise 13.

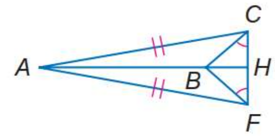
Cross-Curricular Project

You can use properties of triangles to prove Thales of Miletus' important geometric ideas. Visit ca.geometryonline.com to continue work on your project.

EXAMPLE 3 Congruent Segments and Angles

a. Name two congruent angles.

$\angle AFC$ is opposite $\overline{AC}$ and $\angle ACF$ is opposite $\overline{AF}$, so $\angle AFC \cong \angle ACF$.



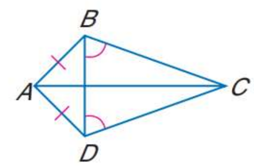
b. Name two congruent segments.

By the converse of the Isosceles Triangle Theorem, the sides opposite congruent angles are congruent. So, $\overline{BC} \cong \overline{BF}$.

CHECK Your Progress

3A. Name two congruent angles.

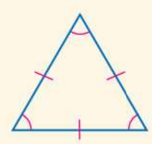
3B. Name two congruent segments.



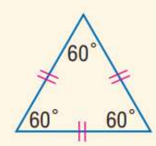
Properties of Equilateral Triangles Recall that an equilateral triangle has three congruent sides. The Isosceles Triangle Theorem leads to two corollaries about the angles of an equilateral triangle.

COROLLARIES

4.3 A triangle is equilateral if and only if it is equiangular.



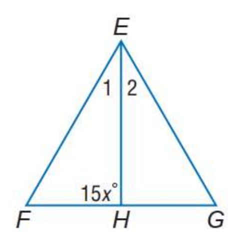
4.4 Each angle of an equilateral triangle measures 60°.



You will prove Corollaries 4.3 and 4.4 in Exercises 11 and 12.

EXAMPLE 4 Use Properties of Equilateral Triangles

$\triangle EFG$ is equilateral, and $\overline{EH}$ bisects $\angle E$.



a. Find $m\angle 1$ and $m\angle 2$.

Each angle of an equilateral triangle measures 60°. So, $m\angle 1 + m\angle 2 = 60$. Since the angle was bisected, $m\angle 1 = m\angle 2$. Thus, $m\angle 1 = m\angle 2 = 30$.

b. ALGEBRA Find x.

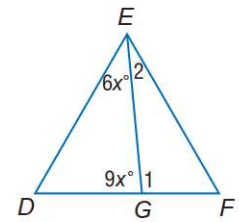
$m\angle EFH + m\angle 1 + m\angle EHF = 180$	Angle Sum Theorem
$60 + 30 + 15x = 180$	$m\angle EFH = 60$, $m\angle 1 = 30$, $m\angle EHF = 15x$
$90 + 15x = 180$	Add.
$15x = 90$	Subtract 90 from each side.
$x = 6$	Divide each side by 15.

CHECK Your Progress

△*DEF* is equilateral.

4A. Find x.

4B. Find $m\angle 1$ and $m\angle 2$.

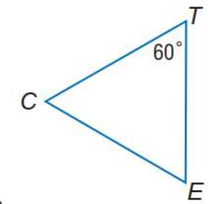


CHECK Your Understanding

Examples 1, 4 (pp. 245, 247)

PROOF **Write a two-column proof.**

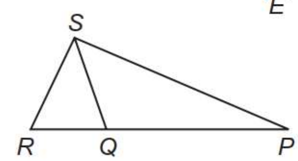
1. **Given:** $\triangle CTE$ is isosceles with vertex $\angle C$.
$m\angle T = 60$
Prove: $\triangle CTE$ is equilateral.



Example 2 (p. 246)

2. **STANDARDS PRACTICE** If $\overline{PQ} \cong \overline{QS}$, $\overline{QR} \cong \overline{RS}$, and $m\angle PRS = 72$, what is $m\angle QPS$?

A 27 **B** 54 **C** 63 **D** 72

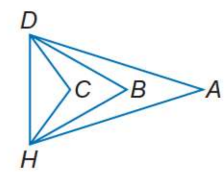


Example 3 (p. 247)

Refer to the figure.

3. If $\overline{AD} \cong \overline{AH}$, name two congruent angles.

4. If $\angle BDH \cong \angle BHD$, name two congruent segments.



Exercises

HOMEWORK HELP	
For Exercises	See Examples
5–10	3
11–13	1
14, 15	4
37, 38	2

Refer to the figure for Exercises 5–10.

5. If $\overline{LT} \cong \overline{LR}$, name two congruent angles.

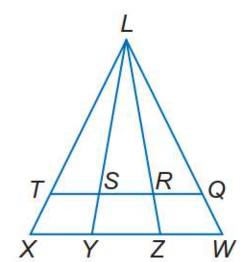
6. If $\overline{LX} \cong \overline{LW}$, name two congruent angles.

7. If $\overline{SL} \cong \overline{QL}$, name two congruent angles.

8. If $\angle LXY \cong \angle LYX$, name two congruent segments.

9. If $\angle LSR \cong \angle LRS$, name two congruent segments.

10. If $\angle LYW \cong \angle LWY$, name two congruent segments.



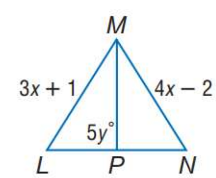
PROOF **Write a two-column proof.**

11. Corollary 4.3 **12.** Corollary 4.4 **13.** Theorem 4.10

Triangle *LMN* is equilateral, and $\overline{MP}$ bisects $\overline{LN}$.

14. Find x and y.

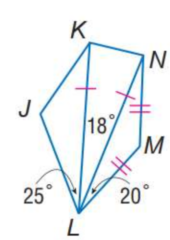
15. Find the measure of each side.



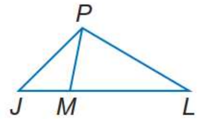
△*KLN* and △*LMN* are isosceles and $m\angle JKN = 130$. Find each measure.

16. $m\angle LNM$ **17.** $m\angle M$

18. $m\angle LKN$ **19.** $m\angle J$



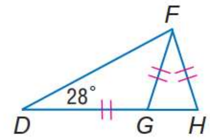
In the figure, $\overline{JM} \cong \overline{PM}$ and $\overline{ML} \cong \overline{PL}$.



20. If $m\angle PLJ = 34$, find $m\angle JPM$.

21. If $m\angle PLJ = 58$, find $m\angle PJL$.

$\triangle DFG$ and $\triangle FGH$ are isosceles, $m\angle FDH = 28$, and $\overline{DG} \cong \overline{FG} \cong \overline{FH}$. Find each measure.



22. $m\angle DFG$

23. $m\angle DGF$

24. $m\angle FGH$

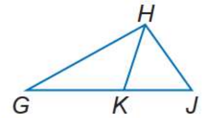
25. $m\angle GFH$

Real-World Link

Spaceship Earth is a completely spherical geodesic dome that is covered with 11,324 triangular aluminum and plastic alloy panels.

Source: disneyworld.disney.go.com

In the figure, $\overline{GK} \cong \overline{GH}$ and $\overline{HK} \cong \overline{KJ}$.

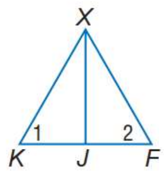


26. If $m\angle HGK = 28$, find $m\angle HJK$.

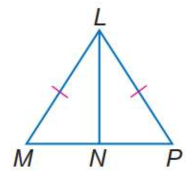
27. If $m\angle HGK = 42$, find $m\angle HKJ$.

PROOF **Write a two-column proof for each of the following.**

28. Given: $\triangle XKF$ is equilateral.
$\overline{XJ}$ bisects $\angle X$.
Prove: J is the midpoint of $\overline{KF}$.



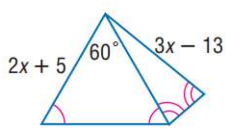
29. Given: $\triangle MLP$ is isosceles.
N is the midpoint of $\overline{MP}$.
Prove: $\overline{LN} \perp \overline{MP}$



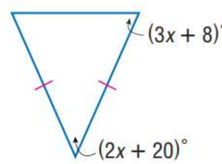
30. DESIGN The exterior of Spaceship Earth at Epcot Center in Orlando, Florida, is made up of triangles. Describe the minimum requirement to show that these triangles are equilateral.

ALGEBRA **Find x.**

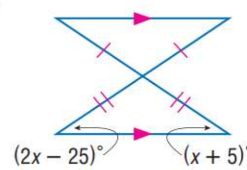
31.



32.



33.

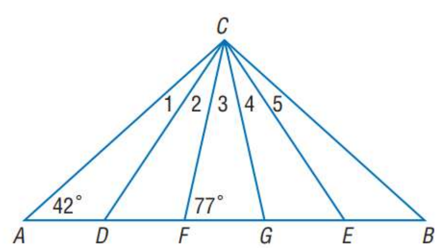


EXTRA PRACTICE
See pages 808, 831.
Math Online
Self-Check Quiz at
ca.geometryonline.com

H.O.T. Problems

34. OPEN ENDED Describe a method to construct an equilateral triangle.

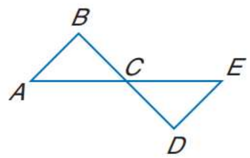
35. CHALLENGE In the figure, $\triangle ABC$ is isosceles, $\triangle DCE$ is equilateral, and $\triangle FCG$ is isosceles. Find the measures of the five numbered angles at vertex C.



36. *Writing in Math* Explain how triangles can be used in art. Describe at least three other geometric shapes and how they are used in art. Include an interpretation of how and why isosceles triangles are used in the painting shown at the beginning of the lesson.

STANDARDS PRACTICE

37. In the figure below, $\overline{AE}$ and $\overline{BD}$ bisect each other at point *C*.



Which additional piece of information would be enough to prove that $\overline{CD} \cong \overline{DE}$?

A $\angle A \cong \angle C$ **C** $\angle ACB \cong \angle EDC$

B $\angle B \cong \angle D$ **D** $\angle A \cong \angle B$

38. REVIEW What quantity should be added to both sides of this equation to complete the square?

$$x^2 - 10x = 3$$

F -25

G -5

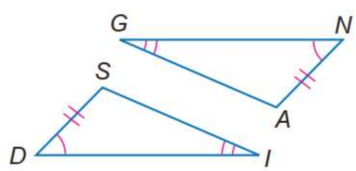
H 5

J 25

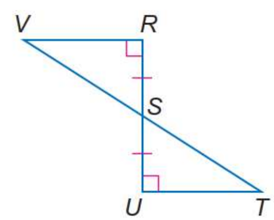
Spiral Review

PROOF Write a paragraph proof. (Lesson 4-5)

39. Given: $\angle N \cong \angle D$, $\angle G \cong \angle I$, $\overline{AN} \cong \overline{SD}$
Prove: $\triangle ANG \cong \triangle SDI$



40. Given: $\overline{VR} \perp \overline{RS}$, $\overline{UT} \perp \overline{SU}$, $\overline{RS} \cong \overline{US}$
Prove: $\triangle VRS \cong \triangle TUS$



Determine whether $\triangle QRS \cong \triangle EGH$ given the coordinates of the vertices. Explain. (Lesson 4-4)

41. $Q(-3, 1)$, $R(1, 2)$, $S(-1, -2)$, $E(6, -2)$, $G(2, -3)$, $H(4, 1)$

42. $Q(1, -5)$, $R(5, 1)$, $S(4, 0)$, $E(-4, -3)$, $G(-1, 2)$, $H(2, 1)$

43. LANDSCAPING Lucas is drawing plans for a client's backyard on graph paper. The client wants two perpendicular pathways to cross at the center of her backyard. If the center of the backyard is set at (0, 0) and the first path goes from one corner of the backyard at $(-6, 12)$ to the other corner at $(6, -12)$, at what coordinates will the second path begin and end? (Lesson 3-3)

Construct a truth table for each compound statement. (Lesson 2-2)

44. *a* and *b* **45.** $\sim p$ or $\sim q$ **46.** *k* and $\sim m$ **47.** $\sim y$ or *z*

GET READY for the Next Lesson

PREREQUISITE SKILL Find the coordinates of the midpoint of the segment with endpoints that are given. (Lesson 1-3)

48. $A(2, 15)$, $B(7, 9)$ **49.** $C(-4, 6)$, $D(2, -12)$ **50.** $E(3, 2.5)$, $F(7.5, 4)$