CHAPTER 8

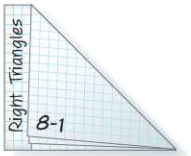
Study Guide and Review

Download Vocabulary Review from ca.geometryonline.com

FOLDABLES Study Organizer

GET READY to Study

Be sure the following Key Concepts are noted in your Foldable.



Key Concepts

Geometric Mean (Lesson 8-1)

- For two positive numbers a and b, the geometric mean is the positive number x where the proportion $a : x = x : b$ is true. This proportion can be written using fractions as $\frac{a}{x} = \frac{x}{b}$ or with cross products as $x^2 = ab$ or $x = \sqrt{ab}$.

Pythagorean Theorem (Lesson 8-2)

- In a right triangle, the sum of the squares of the measures of the legs equals the square of the hypotenuse.

Special Right Triangles (Lesson 8-3)

- The measures of the sides of a 45°–45°–90° triangle are x, x, and $x\sqrt{2}$.
- The measures of the sides of a 30°–60°–90° triangle are x, $x\sqrt{3}$, and $2x$.

Trigonometry (Lesson 8-4)

- Trigonometric Ratios:

$$\sin A = \frac{\text{opposite leg}}{\text{hypotenuse}}$$

$$\cos A = \frac{\text{adjacent leg}}{\text{hypotenuse}}$$

$$\tan A = \frac{\text{opposite leg}}{\text{adjacent leg}}$$

Laws of Sines and Cosines (Lessons 8-6 and 8-7)

Let $\triangle ABC$ be any triangle with a, b, and c representing the measures of the sides opposite the angles with measures A, B, and C, respectively.

- Law of Sines: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$
- Law of Cosines: $a^2 = b^2 + c^2 - 2bc \cos A$
  $b^2 = a^2 + c^2 - 2ac \cos B$
  $c^2 = a^2 + b^2 - 2ab \cos C$

Key Vocabulary

angle of depression (p. 465)	sine (p. 456)
angle of elevation (p. 464)	solving a triangle (p. 472)
cosine (p. 456)	tangent (p. 456)
geometric mean (p. 432)	trigonometric ratio (p. 456)
Pythagorean triple (p. 443)	trigonometry (p. 456)

Vocabulary Check

State whether each sentence is *true* or *false*. If *false*, replace the underlined word or number to make a *true* sentence.

1. To solve a triangle means to find the measures of all its sides and angles.
2. The Law of Sines can be applied if you know the measures of two sides and an angle opposite one of these sides of the triangle.
3. In any triangle, the sum of the squares of the measures of the legs equals the square of the measure of the hypotenuse.
4. An angle of depression is the angle between the line of sight and the horizontal when an observer looks upward.
5. The geometric mean between two numbers is the positive square root of their product.
6. A 30°-60°-90° triangle is isosceles.
7. Looking at a city while flying in a plane is an example that uses an angle of elevation.
8. The numbers 3, 4, and 5 form a Pythagorean identity.

Vocabulary Review at ca.geometryonline.com

Lesson-by-Lesson Review

8-1 Geometric Mean (pp. 432–438)

Find the geometric mean between each pair of numbers.

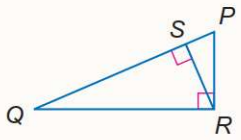
9. 4 and 16

10. 4 and 81

11. 20 and 35

12. 18 and 44

13. In PQR, $PS = 8$, and $QS = 14$. Find RS.



14. INDIRECT MEASUREMENT To estimate the height of the Space Needle in Seattle, Washington, James held a book up to his eyes so that the top and bottom of the building were in line with the bottom edge and binding of the cover. If James' eye level is 6 feet from the ground and he is standing 60 feet from the tower, how tall is the tower?

Example 1 **Find the geometric mean between 10 and 30.**

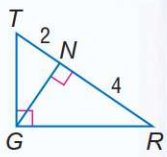
$\frac{10}{x} = \frac{x}{30}$ Definition of geometric mean

$x^2 = 300$ Cross products

$x = \sqrt{300}$ or $10\sqrt{3}$ Simplify.

Example 2 **Find NG in $\triangle TGR$.**

The measure of the altitude is the geometric mean between the measures of the two hypotenuse segments.



$\frac{TN}{GN} = \frac{GN}{RN}$ Definition of geometric mean

$\frac{2}{GN} = \frac{GN}{4}$ $TN = 2$, $RN = 4$

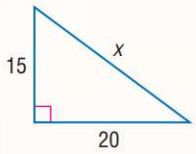
$8 = (GN)^2$ Cross products

$\sqrt{8}$ or $2\sqrt{2} = GN$ Take the square root of each side.

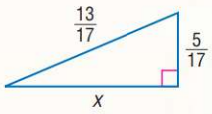
8-2 The Pythagorean Theorem and Its Converse (pp. 440–446)

Find x.

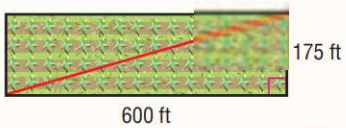
15.



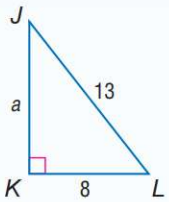
16.



17. FARMING A farmer wishes to create a maze in his corn field. He cuts a path 625 feet across the diagonal of the rectangular field. Did the farmer create two right triangles? Explain.



Example 3 **Use $\triangle JKL$ to find a.**



$a^2 + (LK)^2 = (JL)^2$ Pythagorean Theorem

$a^2 + 8^2 = 13^2$ $LK = 8$ and $JL = 13$

$a^2 + 64 = 169$ Simplify.

$a^2 = 105$ Subtract 64 from each side.

$a = \sqrt{105}$ Take the square root of each side.

$a \approx 10.2$ Use a calculator.

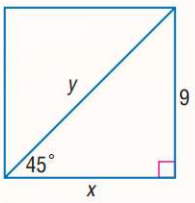
CHAPTER 8

Study Guide and Review

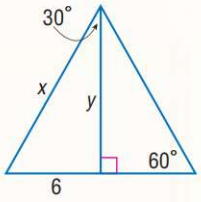
8-3 Special Right Triangles (pp. 448–454)

Find x and y.

18.



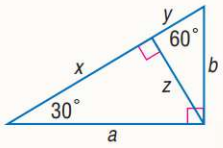
19.



For Exercises 20 and 21, use the figure.

20. If $y = 18$, find z and a.

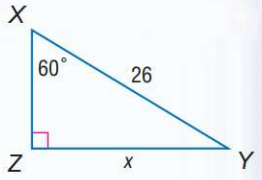
21. If $x = 14$, find a, z, b, and y.



22. ORIGAMI To create a bird, Michelle first folded a square piece of origami paper along one of the diagonals. If the diagonal measured 8 centimeters, find the length of one side of the square.

Example 4 **Find x.**

The shorter leg, $\overline{XZ}$, of $\triangle XYZ$ is half the measure of the hypotenuse $\overline{XY}$.

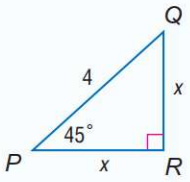


Therefore, $XZ = \frac{1}{2}(26)$ or 13. The longer leg is $\sqrt{3}$ times the measure of the shorter leg.

So, $x = 13\sqrt{3}$.

Example 5 **Find x.**

The hypotenuse of a 45°-45°-90° triangle is $\sqrt{2}$ times the length of a leg.



$$x\sqrt{2} = 4$$

$$x = \frac{4}{\sqrt{2}}$$

$$x = \frac{4}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \text{ or } 2\sqrt{2}$$

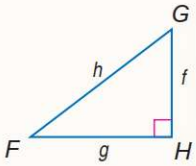
8-4 Trigonometry (pp. 456–462)

Use $\triangle FGH$ to find sin F, cos F, tan F, sin G, cos G, and tan G. Express each ratio as a fraction and as a decimal to the nearest hundredth.

23. $f = 9, g = 12, h = 15$

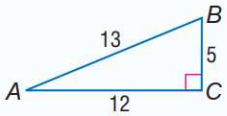
24. $f = 7, g = 24, h = 25$

25. $f = 9, g = 40, h = 41$



26. SPACE FLIGHT A space shuttle is directed towards the Moon but drifts 0.8° from its calculated path. If the distance from Earth to the Moon is 240,000 miles, how far has the space shuttle drifted from its path when it reaches the Moon?

Example 6 **Find sin A, cos A, and tan A. Express as a fraction and as a decimal.**



$$\sin A = \frac{\text{opposite leg}}{\text{hypotenuse}} = \frac{BC}{AB} = \frac{5}{13} \text{ or about } 0.38$$

$$\cos A = \frac{\text{adjacent leg}}{\text{hypotenuse}} = \frac{AC}{AB} = \frac{12}{13} \text{ or about } 0.92$$

$$\tan A = \frac{\text{opposite leg}}{\text{adjacent leg}} = \frac{BC}{AC} = \frac{5}{12} \text{ or about } 0.42$$

Mixed Problem Solving
For mixed problem-solving practice, see page 835.

8-5 Angles of Elevation and Depression (pp. 464–470)

Determine the angles of elevation or depression in each situation.

27. Upon takeoff, an airplane must clear a 60-foot pole at the end of a runway 500 yards long.

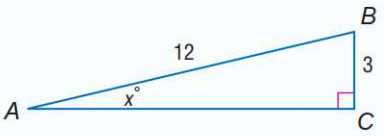
28. An escalator descends 100 feet for each horizontal distance of 240 feet.

29. A hot-air balloon ascends 50 feet for every 1000 feet traveled horizontally.

30. EAGLES An eagle, 1350 feet in the air, notices a rabbit on the ground. If the horizontal distance between the eagle and the rabbit is 700 feet, at what angle of depression must the eagle swoop down to catch the rabbit and fly in a straight path?

Example 7 **The ramp of a loading dock measures 12 feet and has a height of 3 feet. What is the angle of elevation?**

Make a drawing.



Let x represent $m\angle BAC$.

$\sin x^\circ = \frac{BC}{AB}$ $\quad \sin x = \frac{\text{opposite leg}}{\text{hypotenuse}}$

$\sin x^\circ = \frac{3}{12}$ $\quad BC = 3$ and $AB = 12$

$x = \sin^{-1}\left(\frac{3}{12}\right)$ $\quad$ Find the inverse.

$x \approx 14.5$ $\quad$ Use a calculator.

The angle of elevation for the ramp is about 14.5°.

8-6 The Law of Sines (pp. 471–477)

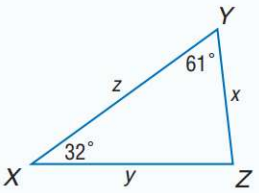
Find each measure using the given measures of $\triangle FGH$. Round angle measures to the nearest degree and side measures to the nearest tenth.

31. Find f if $g = 16$, $m\angle G = 48$, and $m\angle F = 82$.

32. Find $m\angle H$ if $h = 10.5$, $g = 13$, and $m\angle G = 65$.

33. GARDENING Elena is planning a triangular garden. She wants to build a fence around the garden to keep out the deer. The length of one side of the garden is 26 feet. If the angles at the end of this side are 78° and 44°, find the length of fence needed to enclose the garden.

Example 8 **Find x if $y = 15$. Round to the nearest tenth.**



To find x and z, use proportions involving $\sin Y$ and y.

$\frac{\sin Y}{y} = \frac{\sin X}{x}$ $\quad$ Law of Sines

$\frac{\sin 61^\circ}{15} = \frac{\sin 32^\circ}{x}$ $\quad$ Substitute.

$x \sin 61^\circ = 15 \sin 32^\circ$ $\quad$ Cross Products

$x = \frac{15 \sin 32^\circ}{\sin 61^\circ}$ $\quad$ Divide.

$x \approx 9.1$ $\quad$ Use a calculator.

8-7 The Law of Cosines (pp. 479–485)

In $\triangle XYZ$, given the following measures, find the measures of the missing side.

34. $x = 7.6, y = 5.4, m\angle Z = 51$

35. $x = 21, m\angle Y = 73, z = 16$

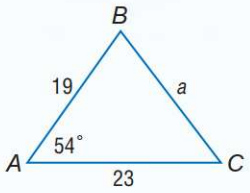
Solve each triangle using the given information. Round angle measures to the nearest degree and side measures to the nearest tenth.

36. $c = 18, b = 13, m\angle A = 64$

37. $b = 5.2, m\angle C = 53, c = 6.7$

38. ART Adelina is creating a piece of art that is in the shape of a parallelogram. Its dimensions are 35 inches by 28 inches and one angle is 80°. Find the lengths of both diagonals.

Example 9 Find a.



$a^2 = b^2 + c^2 - 2bc \cos A$ Law of Cosines

$a^2 = 23^2 + 19^2 - 2(23)(19) \cos 54°$ $b = 23$, $c = 19$, and $m\angle A = 54$

$a^2 = 890 - 874 \cos 54°$ Simplify.

$a = \sqrt{890 - 874 \cos 54°}$ Take the square root of each side.

$a \approx 19.4$ Use a calculator.