****** Example Problem (Detailed Solution)**

A 2 kg cart moving at 3 m/s collides and sticks to a 3 kg cart initially at rest. Find the final velocity of the combined system. Also determine how much kinetic energy was lost during the collision.

Step 1: Identify Type of Collision

They stick together ⇒ **Perfectly inelastic**.

Momentum conserved; kinetic energy not conserved.

Step 2: Known Values

- $m_1 = 2$ kg
- $v_1 = 3$ m/s
- $m_2 = 3$ kg
- $v_2 = 0$ m/s

Step 3: Apply Momentum Conservation

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v_f$$

Substitute:

$$(2)(3) + (3)(0) = (5)v_f$$
$$6 = 5v_f$$
$$v_f = \frac{6}{5} = 1.2 \text{ m/s}$$

Step 4: Compute Initial and Final Kinetic Energy

Initial KE

$$KE_i = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

$$KE_i = \frac{1}{2}(2)(3^2) + 0$$
$$KE_i = 1 \times 9 = 9 \text{ J}$$

Final KE

$$KE_f = \frac{1}{2}(m_1 + m_2)v_f^2$$
$$KE_f = \frac{1}{2}(5)(1.2^2)$$
$$KE_f = \frac{1}{2}(5)(1.44)$$
$$KE_f = 2.5 \times 1.44 = 3.6 \text{ J}$$

Step 5: Energy Lost

$$\Delta KE = KE_i - KE_f$$
$$\Delta KE = 9 - 3.6 = 5.4 \text{ J}$$

Final Answer

- **Final velocity**: **1.2 m/s**
- **Kinetic energy lost**: **5.4 J**
- Energy lost becomes: heat, sound, deformation → reminder of real-world physics.

******* Problem**

A 2.00 kg cart (cart A) is moving right at 4.00 m/s and collides elastically in one dimension with a 3.00 kg cart (cart B) that is initially at rest. Find the final velocities $v_{A,f}$ and $v_{B,f}$. State their directions.

Step 1 — Identify the collision type and givens

- Elastic collision → **momentum and kinetic energy are both conserved.**
- $m_A = 2.00$ kg

- $v_{A,i} = +4.00$ m/s(take right as positive)
- $m_B = 3.00$ kg
- $v_{B,i} = 0.00$ m/s

We will use the standard 1-D elastic collision formulas (they follow directly from conserving momentum and kinetic energy):

$$v_{A,f} = \frac{m_A - m_B}{m_A + m_B}\, v_{A,i} + \frac{2m_B}{m_A + m_B}\, v_{B,i}$$
$$v_{B,f} = \frac{2m_A}{m_A + m_B}\, v_{A,i} + \frac{m_B - m_A}{m_A + m_B}\, v_{B,i}$$

Because $v_{B,i} = 0$, the formulas simplify.

Step 2 — Compute denominators and numerators carefully

First compute the sum $m_A + m_B$:

- $m_A + m_B = 2.00 + 3.00 = 5.00$ kg.

Now compute $m_A - m_B$:

- $m_A - m_B = 2.00 - 3.00 = -1.00$ kg.

And $2m_A$and $2m_B$:

- $2m_A = 2 \times 2.00 = 4.00$ kg.
- $2m_B = 2 \times 3.00 = 6.00$ kg.

Step 3 — Final velocity of A ($v_{A,f}$)

Using the simplified formula (since $v_{B,i} = 0$):

$$v_{A,f} = \frac{m_A - m_B}{m_A + m_B}\, v_{A,i}.$$

Substitute numbers:

$$v_{A,f} = \frac{-1.00}{5.00} \times 4.00.$$

Compute fraction:

$$\frac{-1.00}{5.00} = -0.2000.$$

Multiply:

$$-0.2000 \times 4.00 = -0.8000 \text{ m/s}.$$

So $v_{A,f} = -0.800$ m/s.
Interpretation: negative → cart A reverses direction and moves left at 0.800 m/s.

Step 4 — Final velocity of B ($v_{B,f}$)

Using the simplified formula:

$$v_{B,f} = \frac{2m_A}{m_A + m_B} v_{A,i}.$$

Substitute numbers:

$$v_{B,f} = \frac{4.00}{5.00} \times 4.00.$$

Compute fraction:

$$\frac{4.00}{5.00} = 0.8000.$$

Multiply:

$$0.8000 \times 4.00 = 3.2000 \text{ m/s}.$$

So $v_{B,f} = +3.20$ m/s.
Interpretation: positive → cart B moves right at 3.20 m/s.

Step 5 — Quick checks

Momentum check

Initial momentum:

$$p_i = m_A v_{A,i} + m_B v_{B,i} = (2.00)(4.00) + (3.00)(0.00) = 8.00 + 0.00$$
$$= 8.00 \text{ k} \qquad \text{g\cdotpm/s.}$$

Final momentum:

$$p_f = m_A v_{A,f} + m_B v_{B,f} = (2.00)(-0.8000) + (3.00)(3.2000).$$

Compute terms:

- $(2.00)(-0.8000) = -1.6000$
- $(3.00)(3.2000) = 9.6000$

Sum:

$$p_f = -1.6000 + 9.6000 = 8.0000 \text{ k} \qquad \text{g\cdotpm/s.}$$

Momentum conserved (8.00 → 8.00).

Kinetic energy check

Initial KE:

$$KE_i = \frac{1}{2} m_A v_{A,i}^2 + \frac{1}{2} m_B v_{B,i}^2 = \frac{1}{2}(2.00)(4.00^2) + 0 = 1.00 \times 16.00 = 16.00 \text{ J.}$$

Final KE:

$$KE_f = \frac{1}{2}(2.00)(-0.8000^2) + \frac{1}{2}(3.00)(3.2000^2).$$

Compute squares:

- $(-0.8000)^2 = 0.640000.$
- $3.2000^2 = 10.240000.$

Compute each KE:

- First term: $\frac{1}{2}(2.00)(0.640000) = 1.00 \times 0.640000 = 0.640000$ J.
- Second term: $\frac{1}{2}(3.00)(10.240000) = 1.50 \times 10.240000 = 15.360000$ J.

Sum:

$$KE_f = 0.640000 + 15.360000 = 16.000000 \text{ J.}$$

Kinetic energy conserved (16.00 → 16.00 J): good — confirms elastic collision.

Final answers (clean)

- $v_{A,f} = -0.800$ m/s→ cart A moves **left** at **0.800 m/s**.
- $v_{B,f} = +3.20$ m/s→ cart B moves **right** at **3.20 m/s**.

***** **Problem**

A 1.5 kg ball (Ball A) moves at 6.0 m/s **east** and collides with a 2.0 kg ball (Ball B) moving at 4.0 m/s **north**. After the collision (assume perfectly elastic), ball A moves at 4.0 m/s at 30° north of east. Find ball B's final speed and direction.

Step 1 — Diagram

Before collision:

A → (east)

B ↑ (north)

After collision:

A ↗ at 30°

B = ? (unknown direction)

Step 2 — Components

Initial components

Ball A (east → +x):

- $v_{Ax,i} = +6.0$
- $v_{Ay,i} = 0$

Ball B (north → +y):

- $v_{Bx,i} = 0$
- $v_{By,i} = +4.0$

Final components of ball A

$$v_{A,f} = 4.0 \text{ m/s}, \theta = 30^\circ$$
$$v_{Ax,f} = 4\cos\ 30^\circ = 4(0.866) = 3.464$$
$$v_{Ay,f} = 4\sin\ 30^\circ = 4(0.5) = 2.0$$

Ball B final velocity = unknown $v_{Bx,f}, v_{By,f}$

Step 3 — Apply momentum conservation

X-direction

$$m_A v_{Ax,i} + m_B v_{Bx,i} = m_A v_{Ax,f} + m_B v_{Bx,f}$$

Substitute:

$$1.5(6.0) + 2.0(0) = 1.5(3.464) + 2.0(v_{Bx,f})$$

Compute left:
1.5 × 6 = 9.0

Compute right:
1.5 × 3.464 = 5.196

Equation:

$$9.0 = 5.196 + 2v_{Bx,f}$$
$$2v_{Bx,f} = 3.804$$

$$v_{Bx,f} = 1.902 \text{ m/s}$$

Y-direction

$$m_A v_{Ay,i} + m_B v_{By,i} = m_A v_{Ay,f} + m_B v_{By,f}$$

Substitute:

$$1.5(0) + 2.0(4.0) = 1.5(2.0) + 2.0(v_{By,f})$$

Left side:
2 × 4 = 8

Right side first term:
1.5 × 2 = 3

Equation:

$$8 = 3 + 2v_{By,f}$$
$$2v_{By,f} = 5$$
$$v_{By,f} = 2.50 \text{ m/s}$$

Step 4 — Final speed and angle of Ball B

Speed:

$$v_{B,f} = \sqrt{v_{Bx,f}^2 + v_{By,f}^2}$$
$$v_{B,f} = \sqrt{(1.902)^2 + (2.50)^2}$$
$$v_{B,f} = \sqrt{3.618 + 6.25} = \sqrt{9.868} = 3.14 \text{ m/s}$$

Angle relative to +x (east):

$$\theta = \tan^{-1}(\frac{v_{By,f}}{v_{Bx,f}})$$
$$\theta = \tan^{-1}(\frac{2.50}{1.902}) = 52.3°$$

Final Answer

Ball B moves at:

$$\boxed{3.14 \text{ m/s}, \quad 52^\circ \text{ north of east}}$$

******* Hard 1D *Inelastic* Collision Problem (objects stick for part of the motion)**

A 3.0 kg projectile moving at **25 m/s** embeds itself partially into a 6.0 kg wooden block at rest. After traveling *together* for a short time, the projectile separates from the block and continues forward at **5.0 m/s**, while the block continues moving forward as well.

Assume the only perfectly inelastic part is the initial sticking.
No external horizontal forces act.

(a) Determine the common velocity of the block + projectile just after impact (before separation).
(b) Determine the final velocity of the block after the projectile separates.
(c) Compute the total kinetic energy lost in the collision.
(d) Explain physically why the projectile slows down so drastically.

Problem restatement (numbers)

- Projectile mass: $m_p = 3.0$ kg
- Projectile initial speed: $v_{p,i} = 25.0$ m/s(to the right)
- Block mass: $m_b = 6.0$ kg
- Block initial speed: $v_{b,i} = 0.0$ m/s
- After sticking briefly they move together, then the projectile separates and thereafter the projectile's final speed is $v_{p,f} = 5.0$ m/s(to the right).
- No external horizontal forces.

We will find:
(a) common velocity just after the initial (perfectly) inelastic sticking,

(b) final velocity of the block after separation,
(c) kinetic energy lost (both during sticking and net change from initial → final),
(d) physical explanation.

(a) Common velocity just after the initial (sticking) impact

Use conservation of linear momentum for the instant immediately **after** the projectile embeds (before any separation). Momentum before impact equals momentum of the combined mass after sticking.

Initial total momentum:

$$p_i = m_p v_{p,i} + m_b v_{b,i} = (3.0)(25.0) + (6.0)(0.0).$$

Compute each term:

- $3.0 \times 25.0 = 75.0$ k g\cdotpm/s.
- $6.0 \times 0.0 = 0.0$ k g\cdotpm/s.

So $p_i = 75.0 + 0.0 = 75.0$ k g\cdotpm/s.

After sticking, total mass $m_{\text{tot}} = m_p + m_b = 3.0 + 6.0 = 9.0$ kg.

Let v_{com} be the common velocity right after sticking. Momentum conservation:

$$m_{\text{tot}} v_{\text{com}} = p_i.$$

So

$$v_{\text{com}} = \frac{p_i}{m_{\text{tot}}} = \frac{75.0}{9.0}.$$

Compute:

$$\frac{75.0}{9.0} = 8.333 \ldots \text{ m/s}.$$

Round nicely for class: $v_{\text{com}} = 8.33$ m/s(to 3 s.f.).
(Exact fractional form $75/9 = 25/3 \approx 8.333$.)

Answer (a): $\boxed{v_{\text{com}} = 8.33\text{ m/s (to the right)}}$.

(b) Final velocity of the block after the projectile separates

Separation is an internal process — no external horizontal impulse — so **total horizontal momentum is conserved** through the separation event. Use momentum conservation between the instant *just before* separation (both moving together at v_{com}) and the instant *after* separation (projectile at $v_{p,f} = 5.0$and block at unknown $v_{b,f}$).

Total momentum just before separation:

$$p_{\text{before sep}} = (m_p + m_b)\, v_{\text{com}} = 9.0 \times 8.333 \ldots$$
$$= 75.0\text{ k} \qquad \text{g\cdotpm/s.}$$

(Which matches original momentum — good consistency check.)

After separation:

$$p_{\text{after sep}} = m_p v_{p,f} + m_b v_{b,f}.$$

Set equal and solve for $v_{b,f}$:

$$m_p v_{p,f} + m_b v_{b,f} = 75.0$$
$$v_{b,f} = \frac{75.0 - m_p v_{p,f}}{m_b}.$$

Plug numbers:

- $m_p v_{p,f} = 3.0 \times 5.0 = 15.0$ k g\cdotpm/s.
- Numerator: $75.0 - 15.0 = 60.0$ k g\cdotpm/s.
- Divide by $m_b = 6.0$:

$$v_{b,f} = \frac{60.0}{6.0} = 10.0\text{ m/s.}$$

Answer (b): $\boxed{v_{b,f} = 10.0\text{ m/s (to the right)}}$.

(Notice the block ends up faster (10.0 m/s) than the combined speed before separation (8.33 m/s) because the projectile slowed to 5.0 m/s — momentum redistribution.)

(c) Kinetic energy calculations — how much is lost?

We will compute:

- KE before any collision: KE_i(initial projectile + block at rest),
- KE immediately after sticking: KE_{com},
- KE after separation (final): KE_f.

Initial kinetic energy KE_i

$$KE_i = \frac{1}{2} m_p v_{p,i}^2 + \frac{1}{2} m_b v_{b,i}^2.$$

Plug values:

- $\frac{1}{2} m_p v_{p,i}^2 = 0.5 \times 3.0 \times (25.0)^2$.
  Compute:
- $(25.0)^2 = 625.0$.
- $0.5 \times 3.0 = 1.5$.
- So term = $1.5 \times 625.0 = 937.5$ J.

Block initial term is zero. So

$$KE_i = 937.5 \text{ J}.$$

Kinetic energy just after sticking KE_{com}

Combined mass $= 9.0$kg, speed $= 8.333\ldots$m/s.

$$KE_{\text{com}} = \frac{1}{2}(m_p + m_b) v_{\text{com}}^2 = 0.5 \times 9.0 \times (8.333\ldots)^2.$$

Compute v_{com}^2:

$$8.333\ldots^2 = (25/3)^2 = 625/9 \approx 69.444\ldots$$

Now

$$0.5 \times 9.0 = 4.5.$$

So

$$KE_{\text{com}} = 4.5 \times 69.444 \ldots = 312.5 \text{ J}.$$

(Exact: $4.5 \times 625/9 = (4.5 \times 625)/9 = (2812.5)/9 = 312.5$.)

KE lost during the initial sticking event:

$$\Delta KE_{\text{stick}} = KE_i - KE_{\text{com}} = 937.5 - 312.5 = 625.0 \text{ J}.$$

So **625.0 J** of kinetic energy was dissipated (to heat, deformation, sound, etc.) during the inelastic embedding.

Kinetic energy after separation KE_f

After separation we have the projectile at 5.0 m/s and the block at 10.0 m/s:

$$KE_f = \frac{1}{2} m_p v_{p,f}^2 + \frac{1}{2} m_b v_{b,f}^2.$$

Compute termwise:

- Projectile: $0.5 \times 3.0 \times (5.0)^2 = 1.5 \times 25.0 = 37.5$ J.
- Block: $0.5 \times 6.0 \times (10.0)^2 = 3.0 \times 100.0 = 300.0$ J.

So

$$KE_f = 37.5 + 300.0 = 337.5 \text{ J}.$$

Net kinetic energy change from initial → final

$$\Delta KE_{\text{net}} = KE_f - KE_i = 337.5 - 937.5 = -600.0 \text{ J}.$$

So the entire process (initial impact + later separation) results in a net **loss of 600.0 J** of mechanical kinetic energy relative to the starting state.

Energy bookkeeping observation

- KE before impact: **937.5 J**
- KE just after sticking: **312.5 J** (lost **625.0 J**)
- KE after separation: **337.5 J** (gains back **25.0 J** compared with the stuck state)
- Net change from start → finish: **−600.0 J**

That small increase from 312.5 J → 337.5 J (i.e., +25.0 J) during separation means some **stored internal energy** (for example elastic energy, chemical energy, or a propulsive release) was converted back into kinetic energy at the moment of separation. But overall, compared to the original projectile energy, much energy was dissipated.

(d) Physical explanation: why does the projectile slow so drastically?

Intuitive picture:

- The projectile (3.0 kg) is fast (25.0 m/s) and hits a heavier block (6.0 kg) at rest.
- When it embeds, momentum is shared across the **combined heavier mass** (9.0 kg), so the combined speed drops to the common 8.33 m/s. That large reduction is simply the consequence of momentum being distributed over more mass.
- Later, during separation, the projectile emerges at only 5.0 m/s — even less than the combined speed. That means some internal interaction during the sticking/separation cycle transferred additional momentum to the block so the block leaves faster (10.0 m/s) while the projectile is slowed further.
- The big picture: in the initial inelastic embedding, a large amount of kinetic energy is converted into internal forms (heat, deformation, sound). Some of that stored energy can later be released during separation (giving back a little kinetic energy, here +25.0 J), but not nearly enough to restore the original energy because the initial embedding dissipated much more (625.0 J).